



FORCES IN 3D (FORCES IN SPACE)

Moment of a Force:

Moment of Force about a point

The moment  $\vec{M}$  of a force  $\vec{F}$  with respect to a point  $O$  is the cross product  $\vec{r} \times \vec{F}$  (Not  $\vec{F} \times \vec{r}$ ) where  $\vec{r}$  is the position vector relative to point  $O$ , at any point  $P$  on the line of action of  $\vec{F}$ .

\* Moment  $\vec{M}$  represents the tendency of the force  $\vec{F}$ , to rotate a body on which it acts, about an axis called moment axis.

\* This moment axis is passing through  $O$  & perpendicular to the plane containing the force  $\vec{F}$  & the position vector  $\vec{r}$ .

\* The moment vector is usually denoted by a double line arrow ( $\vec{M}$ ) or by a curved arrow about the moment axis as shown in figure.

If  $\vec{r}$  &  $\vec{F}$  are given,

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M} = M_x\vec{i} + M_y\vec{j} + M_z\vec{k} \quad \text{--- (1)}$$



$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

is it  $\vec{M} = (F_y z - F_z y)\hat{i} + (F_x z - F_z x)\hat{j} + (F_y x - F_x y)\hat{k}$  — (2)

Equating (1) & (2) we get,

$M_x = F_y z - F_z y$   
 $M_y = F_x z - F_z x$   
 $M_z = F_y x - F_x y$

where,  $M_x, M_y$  &  $M_z$  are scalar quantities of  $\vec{M}$  about x, y & z axes through O.

**Magnitude of Moment  $\vec{M}$**   
From the moment vector  $\vec{M} = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$

Magnitude of Moment,  $M = \sqrt{M_x^2 + M_y^2 + M_z^2}$

**Direction of Moment  $\vec{M}$**   
Let moment vector  $\vec{M}$  makes angles  $\phi_x, \phi_y$  &  $\phi_z$  about x, y & z axes.

$\cos \phi_x = \frac{M_x}{M} \rightarrow \phi_x = \cos^{-1} \left( \frac{M_x}{M} \right)$

$\phi_y = \cos^{-1} \left( \frac{M_y}{M} \right)$

$\phi_z = \cos^{-1} \left( \frac{M_z}{M} \right)$



Note: Point P may be taken any where on the line of action of  $\vec{F}$ .

$$\vec{H}_O = \vec{r}_{OA} \times \vec{F} = \vec{r}_{OB} \times \vec{F}$$

In case, if moment about any arbitrary point B, of force  $\vec{F}$  acting at A is required, the relative position vector of A, with respect to B should be used ( $\vec{r}_{A/B}$ )

Case (i)  
when position vector A & B are known,  
$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$
  
$$= (\vec{r}_{OA} - \vec{r}_{OB}) \times \vec{F}$$

Case (ii)  
when coordinates of A & B are known  
$$\vec{M}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$