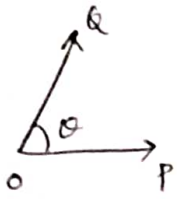
Resultant force of two Concurrent forces : (Analytical)

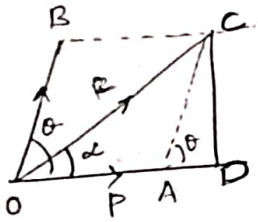
Parallelogram law of forces:

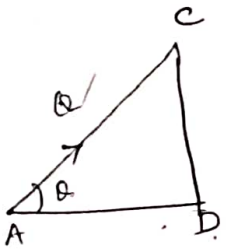
If two forces acting simultaneously at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of that parallelogram originating from that point.

Proof:

P and Q are two concurrent forces acting on point 'O' at an angle of θ as shown. The forces P and Q are graphically represented by the lines OA and OB respectively. The parallelogram OACB is completed by drawing the lines BC and AC parallel to OA and OB resp. In OACB, the diagonal OC represents the resultant forces of P and Q.







In Δ ACD

$$\cos\theta = \frac{AD}{Q} \qquad \sin\theta = \frac{CD}{Q}$$

$$AD = Q\cos\theta \rightarrow (1) \qquad CD = Q\sin\theta \rightarrow (2)$$

$$AD^2 + CD^2 = AC^2 = Q^2 \Rightarrow (3)$$

Δ OCD

$$OC^2 = OD^2 + CD^2$$

$$= (OA + AD)^2 + CD^2$$

$$= OA^2 + AD^2 + 2\cdot OA\cdot AD + CD^2$$

$$OC^2 = OA^2 + (AD^2 + CD^2) + 2\cdot OA\cdot AD$$

$$OC^2 = OA^2 + AC^2 + 2\cdot OA\cdot AD \quad \text{or} \quad R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$