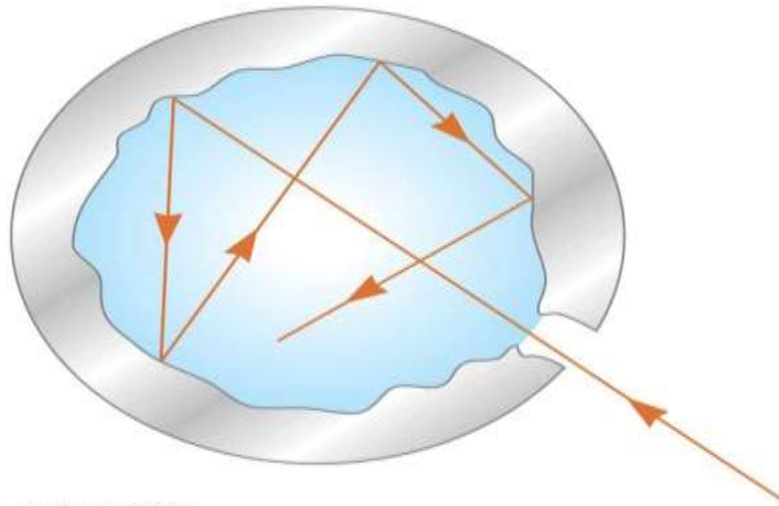


BLACK BODY RADIATION

- **Blackbody**
- a cavity, such as a metal box with a small hole drilled on to it.
- Incoming radiations entering the hole keep bouncing around inside the box with a negligible change of escaping again through the hole => Absorbed, the hole is the ***perfect absorber***



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- When heated, it would emit more radiations from a unit area through the hole at a given temperature => ***perfect emitter***

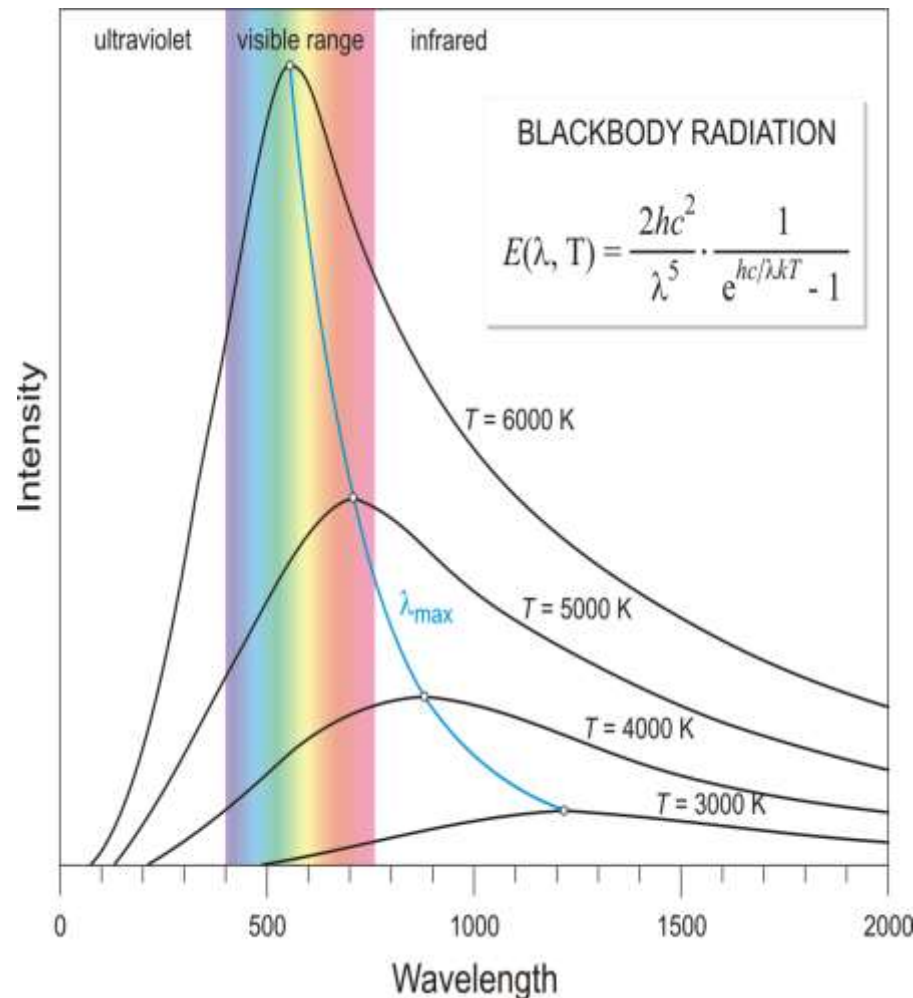
BLACK BODY RADIATION

- A perfect Black body absorbs radiation of all wavelength incident on it. It also emits radiation of all wavelength.
- When Black body is at a higher temperature than its surrounding, then emission is more than absorption.
- The heat radiation emitted by a black body is known as **Black body radiation**.

Theory of Black Body Radiation-Energy spectrum of a black body

The radiation emitted by the black body varies with temperature.

- At a given temp. energy distribution is not continuous.
- The intensity of radiation is maximum at a particular wavelength λ_{\max} .
- With increase in temperature λ_{\max} decreases
- As temp. Increases intensity also increases.
- The area under each spectrum represents the total energy emitted at that particular temp.



Basic definitions

Stefan's Boltzmann's law:

- Total energy emitted at a particular temp. of a object is directly proportional to the fourth power of the temperature of the body. $E \propto T^4$

Wien's displacement law:

The product of wavelength corresponding to maximum intensity λ_{\max} and absolute temp. in a hot body is a constant. $\lambda_{\max} T = \text{Cons.}$

Rayleigh-Jeans law:

Energy distribution is directly proportional to Absolute temp. and inversely proportional to fourth power of wavelength of radiation of hot body.

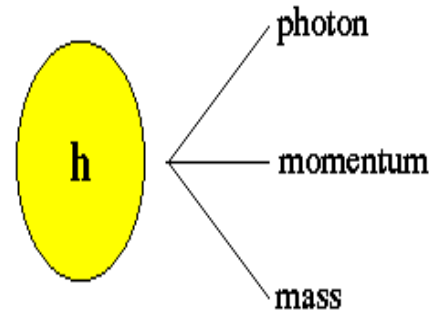
$$E_{\lambda} = \frac{8\pi KT}{\lambda^4}$$

19PB1037 Physics for Engineers
Dr.V.Manikandan / AP/ Physics

PLANK'S THEORY

THE MIND IS
THE MATRIX OF
ALL MATTER
MAX PLANCK

quanta packet



PLANK'S THEORY



PLANCK'S QUANTUM THEORY OF BLACK BODY RADIATION

Planck's theory: [Hypothesis]

A black body contains a **large number of oscillating particles**:

Each particle is vibrating with a characteristic frequency.

The frequency of radiation emitted by the oscillator is the same as the oscillator **frequency**.

The oscillator can absorb energy in multiples of small unit called quanta.

This **quantum of radiation** is **photon**.

The energy of a **photon** is directly proportional to the **frequency of radiation** emitted.

An oscillator **vibrating with frequency** can only emit energy in integral multiples of $h\nu$, where $n = 1, 2, 3, 4, \dots, n$. n is **quantum number**.

PLANCK'S LAW OF RADIATION

The energy density of radiations emitted by a black body at a temperature T in the **wavelength** range λ to $\lambda+d\lambda$ is

$$E_{\lambda} d\lambda = \frac{8\pi h C}{\lambda^5 \left[\exp\left(\frac{hC}{\lambda KT}\right) - 1 \right]} d\lambda$$

$h = 6.625 \times 10^{-34} \text{Js}^{-1}$ - Planck's constant.

$C = 3 \times 10^8 \text{ m/s}$ -velocity of light

$K = 1.38 \times 10^{-23} \text{ J/K}$ -Blotzmann constant

T is the absolute temperature in kelvin.

DERIVATION OF PLANCK'S LAW

Consider a black body with a large number of atomic oscillators. Average energy per oscillator is

$$\bar{E} = \frac{E}{N} \text{ --- --- --- --- (1)}$$

E is the total energy of all the oscillators and N is the number of oscillators.

Let the number of oscillators in ground state is N_0 . According to Maxwell's law of distribution, the number of oscillators having an energy value E_n is

$$N_n = N_0 e^{-\frac{E_n}{kT}} \text{ --- --- --- --- (2)}$$

DERIVATION OF PLANCK'S LAW

T is the absolute temperature. K is the Boltzmann constant.

Let N_0 be the number of oscillators having energy E_0 ,

– N_1 be the number of oscillators having energy E_1 ,

– N_2 be the number of oscillators having energy E_2 and so on.

Then

$$N = N_0 + N_1 + N_2 + \dots(3)$$

$$N = N_0 + N_0 e^{\frac{-E_1}{KT}} + N_0 e^{\frac{-E_2}{KT}} + \dots(4)$$

From Planck's theory, E can take only integral values of $h\nu$.

Hence the possible energy are 0, $h\nu$, $2h\nu$, $3h\nu$ and so on.

$$**i. e. $E_n = nh\nu$, where $n = 0, 1, 2, 3$**$$

DERIVATION OF PLANCK'S LAW

$$E_0 = 0, \quad E_1 = h\nu, \quad E_2 = 2h\nu, \quad E_3 = 3h\nu,$$

$$N = N_0 + N_0 e^{\frac{-h\nu}{KT}} + N_0 e^{\frac{-2h\nu}{KT}} + \dots (5)$$

Taking $x = e^{-\frac{h\nu}{kT}}$ in (5)

$$N = N_0 + N_0 x + N_0 x^2 + N_0 x^3 \dots (6)$$

$$N = N_0 [1 + x + x^2 + x^3 \dots].$$

$$N = \frac{N_0}{(1 - x)} \dots \dots \dots 7) \quad (\text{using Binomial expansion})$$

DERIVATION OF PLANCK'S LAW

The total energy

$$E = E_0 N_0 + E_1 N_1 + E_2 N_2 + E_3 N_3 \dots (8)$$

Substituting the value of $E_0 E_1 E_2 E_3$ etc

$$E = 0 \times N_0 + h\nu N_0 e^{-\frac{h\nu}{kT}} + 2h\nu N_0 e^{-\frac{2h\nu}{kT}} + 3h\nu N_0 e^{-\frac{3h\nu}{kT}} \dots +$$

$$E = h\nu N_0 e^{-\frac{h\nu}{kT}} + 2h\nu N_0 e^{-\frac{2h\nu}{kT}} + \dots \pm \text{---} \text{---} \text{---} \text{---} \text{---} (9)$$

Putting

$$x = e^{-\frac{h\nu}{kT}} \text{ in (8)}$$

$$E = h\nu N_0 x + 2h\nu N_0 x^2 + \dots \pm \text{---} \text{---} \text{---} (10)$$

DERIVATION OF PLANCK'S LAW

$$E = h\nu N_0 [x + 2x^2 + \dots +]$$

$$E = h\nu N_0 x [1 + 2x + \dots +]$$

$$E = \frac{h\nu N_0 x}{(1-x)^2} \text{--- -- -- -- (11)}$$

$$\textit{Since} \left\{ \frac{1}{(1-x)^2} = (1-x)^{-2} = 1 + 2x + \dots \right\}$$

Substituting (11) and (7) in (1)

$$\bar{E} = \frac{\left(\frac{h\nu N_0 x}{(1-x)^2} \right)}{\frac{N_0}{(1-x)}}$$

DERIVATION OF PLANCK'S LAW

$$\bar{E} = \frac{h\nu x}{(1-x)} \quad \bar{E} = \frac{h\nu x}{x\left(\frac{1}{x} - 1\right)} \quad \bar{E} = \frac{h\nu}{\left(\frac{1}{x} - 1\right)}$$

Substituting the value for x

$$\bar{E} = \frac{h\nu}{\left(\frac{1}{\frac{h\nu}{e^{-kT}}} - 1\right)}$$

$$\bar{E} = \frac{h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)} \text{----- (12)}$$

The number of oscillators per unit volume in the wavelength range λ and $\lambda+d\lambda$ is

$$\frac{8\pi d\lambda}{\lambda^4} \text{----- (13)}$$

DERIVATION OF PLANCK'S LAW

Hence the energy density of radiation between the wavelength range λ and $\lambda+d\lambda$ is

$E_\lambda d\lambda =$ No. of oscillator per unit volume in the range λ and $\lambda+d\lambda$ X Average energy.

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^4} X \frac{h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)} \text{ --- (1)}$$

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^4} \frac{hC/\lambda}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$
$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^5} \frac{hC}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^5} \frac{hC}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

DERIVATION OF PLANCK'S LAW

$$E_{\lambda}d\lambda = \frac{8\pi hC}{\lambda^5} \frac{1}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)} d\lambda \text{----- (15)}$$

$$E_{\lambda}d\lambda = \frac{8\pi hC}{\lambda^5 \left[e^{\left(\frac{hc}{\lambda kT}\right)} - 1\right]} d\lambda$$

The equation (16) represents **Planck's law of radiation.**

Planck's law can also be represented in **terms of frequencies.**

$$E_{\nu}d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)} d\nu \text{----- 17}$$