SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT211 – ELECTROMAGNETIC FIELDS

II YEAR/ IV SEMESTER

UNIT 5 – ELECTROMAGNETIC WAVES

TOPIC 5 – DEPTH OF PENETRATION

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

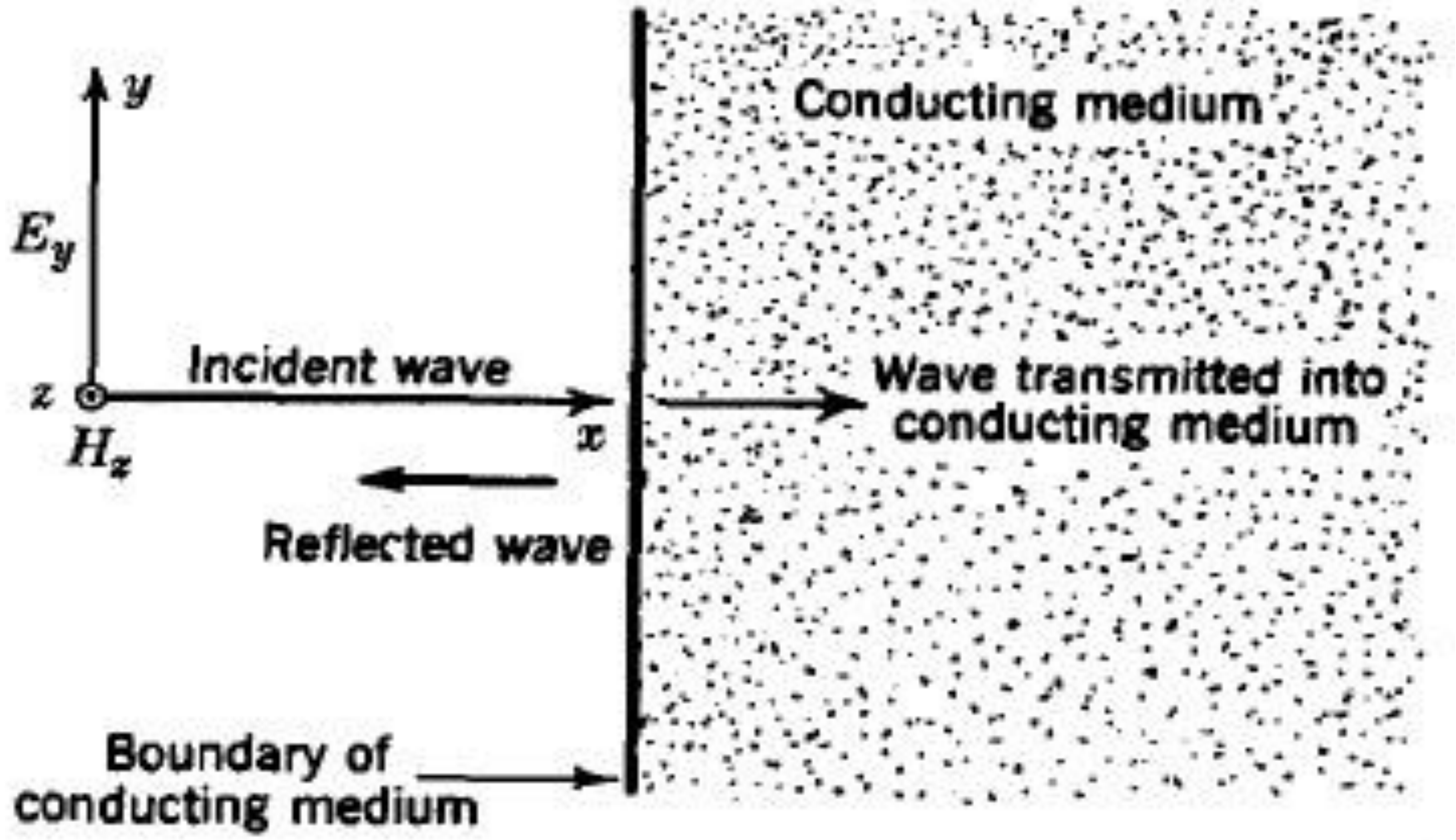

FIGURE 10-15
Plane wave entering conducting medium at normal incidence.

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

10-15, consider the wave that penetrates the conducting medium, i.e., the transmitted wave. Let $x = 0$ at the boundary of the conducting medium, so that x increases positively into the conducting medium.

Let (10-15-14) be written in the form

$$E_y = E_0 e^{-x/\delta} e^{-j(x/\delta)} \qquad (1)$$

where $\delta = \sqrt{2/\omega\mu\sigma}$. At $x = 0$, $E_y = E_0$. This is the amplitude of the field at the surface on the conducting medium. Now δ in (1) has the dimension of distance. At a distance $x = \delta$ the amplitude of the field is

$$|E_y| = E_0 e^{-1} = E_0 \frac{1}{e} \qquad (2)$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Thus, E_y decreases to $1/e$ (36.8 percent) of its initial value, while the wave penetrates to a distance δ. Hence δ is called the $1/e$ *depth of penetration*.

As an example, consider the depth of penetration of a plane electromagnetic wave incident normally on a good conductor, such as copper. Since $\omega = 2\pi f$, the $1/e$ depth becomes

$$\boxed{\delta = \frac{1}{\sqrt{f\pi\mu\sigma}}} \qquad (3)$$

For copper $\mu_r = 1$, so that $\mu = 1.26\ \mu\text{H m}^{-1}$. The conductivity $\sigma = 58\ \text{M℧ m}^{-1}$. Putting these values in (3), we obtain for copper

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

$$\delta = \frac{6.6 \times 10^{-2}}{\sqrt{f}} \qquad (4)$$

where $\delta = 1/e$ depth of penetration, m
f = frequency, Hz

Evaluating (4) at specific frequencies, we find that

At 60 Hz: $\delta = 8.5 \times 10^{-3}$ m

At 1 MHz: $\delta = 6.6 \times 10^{-5}$ m

At 30 GHz: $\delta = 3.8 \times 10^{-7}$ m

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Thus, while at 60 Hz the $1/e$ depth of penetration is 8.5 mm, the penetration decreases in inverse proportion to the square root of the frequency. At 10 mm wavelength (30 GHz) the penetration is only 0.00038 mm, or less than $\frac{1}{2}$ μm. This phenomenon is often called *skin effect*.

Thus, a high-frequency field is damped out as it penetrates a conductor in a shorter distance than a low-frequency field.†

In addition to the $1/e$ depth of penetration, we can speak of other depths for which the electric field decreases to an arbitrary fraction of its original value. For example, consider the depth at which the field is 0.01 (1 percent) of its original value. This depth is obtained by multiplying the $1/e$ depth by 4.6 and may be called the 1 *percent depth of penetration*.

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Phase velocity is given by the ratio ω/β. In the present case, $\beta = 1/\delta$ so that the phase velocity in the conductor is

$$v_c = \omega\delta = \sqrt{\frac{2\omega}{\sigma\mu}} \tag{5}$$

Since the $1/e$ depth is small, the phase velocity in conductors is small. It is apparent from (5) that the velocity is a function of the frequency and hence of the wavelength. In this case, $dv/d\lambda$ is negative, where λ is the free-space wavelength. Hence, conductors are anomalously dispersive media (Sec. 10-7).

The ratio of the velocity of a wave in free space to that in the conducting medium is the index of refraction for the conducting medium. At low frequencies the index for conductors is very large.

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

To find the wavelength λ_c in the conductor, we have from (5) that $f\lambda_c = \omega\delta$, or

$$\lambda_c = 2\pi\delta \tag{6}$$

In (6), both λ_c and δ are in the same units of length. Hence the wavelength in the conductor is 2π times the $1/e$ depth. Since the $1/e$ depth is small for conductors, the wavelength in conductors is small.

Values of the $1/e$ depth, 1 percent depth, wavelength, velocity, and refractive index for a medium of copper are given in Table 10-4 for three frequencies.

It is interesting to note that the electric field is damped to 1 percent of its initial amplitude in about $\frac{3}{4}\lambda$ in the metal.

Since the penetration depth is inversely proportional to the square root of the frequency, a thin sheet of conducting material can act as a low-pass filter for electromagnetic waves.

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Table 10-4 PENETRATION DEPTHS, WAVELENGTH, VELOCITY, AND REFRACTIVE INDEX FOR COPPER

	Frequency		
	60 Hz	1 MHz	30 GHz
Wavelength in free space λ (m)	5 mm	300 m	10 mm
$1/e$ depth, m	8.5×10^{-3}	6.6×10^{-5}	3.8×10^{-7}
1 percent depth, m	3.9×10^{-2}	3×10^{-4}	1.7×10^{-6}
Wavelength in conductor λ_c, m	5.3×10^{-2}	4.1×10^{-4}	2.4×10^{-6}
Velocity in conductor v_c, m s^{-1}	3.2	4.1×10^{2}	7.1×10^{4}
Index of refraction, dimensionless	9.5×10^{7}	7.3×10^{5}	4.2×10^{3}

REFERENCES

- John.D.Kraus , “ Electromagnetics “,5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: “Engineering Electromagnetics” Tata McGraw-Hill, 7th Edition 2007

THANK YOU