17/05/23

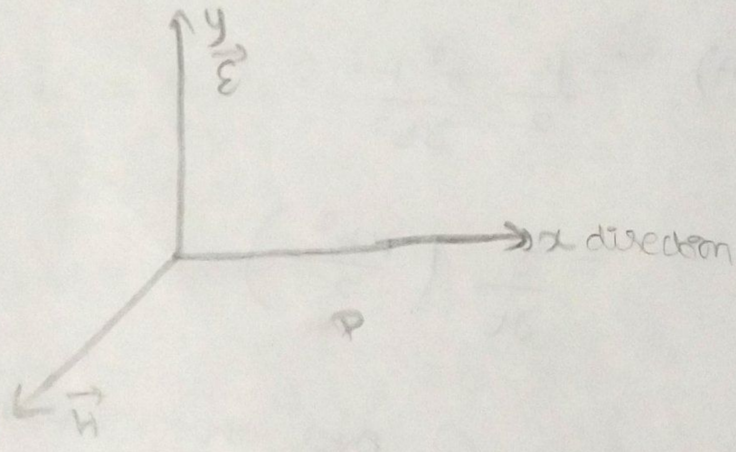
1st period

Unit - V.

Electromagnetic Waves

like spherical wave but r is ||

Derivation of wave equation - Uniform Plane waves :-

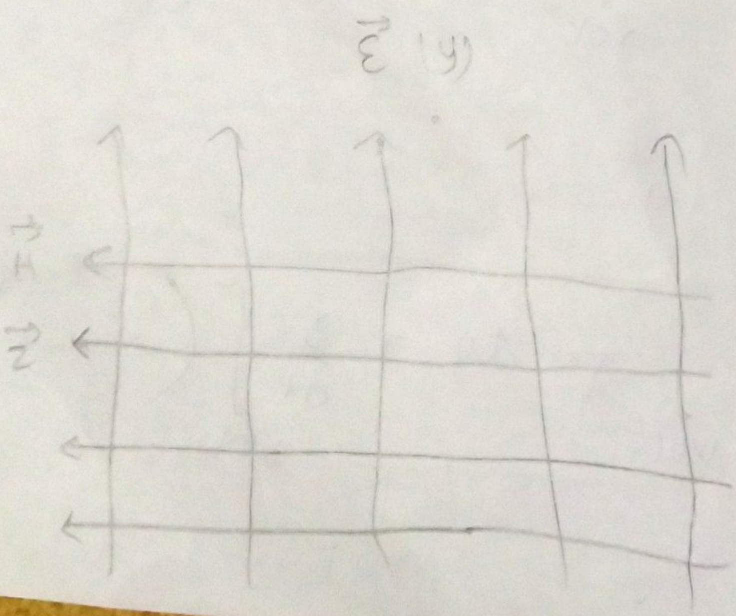


direction of P → wave propagation

$\Rightarrow \vec{E}$ & $\vec{H}$ $\perp^r$ P

* $\vec{E}$ $\perp^r$ $\vec{H}$

* Also called as Transverse Electromagnetic wave.



Polarization
↓
* ~~Polarization~~ orientation of elec. field at any instant of time.

* vertically polarized.

Maxwell's eqn from ampere's Law.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad -①$$

consider, $\sigma = 0, J = 0$ for free space $(J = \sigma E)$.

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$[\because D = \epsilon E]$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} \quad -②$$

Expanding ②

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \frac{\partial}{\partial t}(\hat{x} D_x + \hat{y} D_y + \hat{z} D_z)$$

$$\hat{x}\left[\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y\right] - \hat{y}\left[\frac{\partial}{\partial x} H_z - \frac{\partial}{\partial z} H_x\right] + \hat{z}\left[\frac{\partial}{\partial x} H_y - \frac{\partial H_x}{\partial y}\right]$$

$$= \frac{\partial}{\partial t}(\hat{x} D_x + \hat{y} D_y + \hat{z} D_z)$$

$\because$ the wave is propagating in x direction, the only component contribute is,

$$-\hat{y}\left(\frac{\partial H_z}{\partial x}\right) = \hat{y} \frac{\partial D_y}{\partial t} \quad -③$$

Sub $D = \epsilon E$.

$$\frac{\partial H_z}{\partial x} = -\epsilon \frac{\partial E_y}{\partial t} \quad -④$$

Eqn ④ relates space derivative of H_z to time derivative of E_y.

Maxwell's second eqn from faraday's law.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}\left(\hat{x} B_x + \hat{y} B_y + \hat{z} B_z\right)$$

$$\hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{z}\left(\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x\right)$$

$$= -\frac{\partial}{\partial t}\left(\hat{x} B_x + \hat{y} B_y + \hat{z} B_z\right)$$

Since the wave propagating in x direction, the only component contribute is,

$$\vec{z}\left[\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x\right] = -\hat{z}\frac{\partial B_z}{\partial t} \quad -⑦$$

$$\vec{z}\left[\frac{\partial E_y}{\partial x}\right] = -\hat{z}\frac{\partial B_z}{\partial t}$$

$$\frac{\partial \mathcal{E}_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \quad \text{—(8)}$$

ean (8) relates space derivative of $\mathcal{E}_y$ to time derivative of H_z.

18/05/23
2 period

Differentiating eq (4) wrt time.

$$\underbrace{\frac{\partial}{\partial t}}_{\text{time derivative}} \underbrace{\left(\frac{\partial H_z}{\partial x}\right)}_{\text{space derivative}} = -\varepsilon_r \frac{\partial}{\partial t}\left(\frac{\partial \mathcal{E}_y}{\partial t}\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial H_z}{\partial x}\right) = -\varepsilon_r \frac{\partial^2 \mathcal{E}_y}{\partial t^2} \quad \text{—(9)}$$

diff (8) wrt distance 'x'

$$\frac{\partial}{\partial x}\left(\frac{\partial \mathcal{E}_y}{\partial x}\right) = -\mu \frac{\partial}{\partial x}\left(\frac{\partial H_z}{\partial t}\right)$$

$$\frac{\partial^2 \mathcal{E}_y}{\partial x^2} = -\mu \frac{\partial}{\partial x}\left(\frac{\partial H_z}{\partial t}\right) \quad \text{—(10)}$$

÷ eq (10) by $-\mu$.

$$-\frac{1}{\mu}\left(\frac{\partial^2 \mathcal{E}_y}{\partial x^2}\right) = \frac{\partial}{\partial x}\left(\frac{\partial H_z}{\partial t}\right) \quad \text{—(11)}$$

compare (9) & (11)

LHS of (9) = RHS of (11)

$$-\varepsilon_r \frac{\partial^2 \mathcal{E}_y}{\partial t^2} = -\frac{1}{\mu}\left(\frac{\partial^2 \mathcal{E}_y}{\partial x^2}\right)$$

$$\frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{1}{\mu \varepsilon}\left[\frac{\partial^2 \varepsilon_y}{\partial x^2}\right] \quad \text{— (12)}$$

wave eqn respect to ε_y

Diff eq (6) wrt distance

$$\frac{\partial}{\partial x}\left(\frac{\partial H_z}{\partial x}\right) = -\varepsilon_\varphi \frac{\partial}{\partial x}\left(\frac{\partial \varepsilon_y}{\partial t}\right)$$

$$\frac{\partial^2 H_z}{\partial x^2} = -\varepsilon_\varphi \frac{\partial}{\partial x}\left(\frac{\partial \varepsilon_y}{\partial t}\right) \quad \text{— (13)}$$

Diff eq (8) with respect to time

$$\frac{\partial}{\partial t}\left(\frac{\partial \varepsilon_y}{\partial x}\right) = -\mu \frac{\partial}{\partial t}\left(\frac{\partial H_z}{\partial t}\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \varepsilon_y}{\partial x}\right) = -\mu \frac{\partial^2 H_z}{\partial t^2} \quad \text{— (14)}$$

Divide eq (14) by $-\varepsilon_\varphi$

$$-\varepsilon_\varphi \quad -\frac{1}{\cancel{\varepsilon_\varphi}}\left(\frac{\partial}{\partial t}\right)\left(\frac{\partial \varepsilon_y}{\partial x}\right) = \frac{\varepsilon_\varphi \mu}{1}\frac{\partial^2 H_z}{\partial t^2} \quad \text{— (15)}$$

Compare (13) & (15)

$$\frac{\partial^2 H_z}{\partial x^2} = \varepsilon_\varphi \mu \left(\frac{\partial^2 H_z}{\partial t^2}\right)$$

$$\frac{\partial^2 H_z}{\partial t^2} = \frac{1}{\mu \varepsilon_\varphi}\left[\frac{\partial^2 H_z}{\partial x^2}\right] \quad \text{— (16)}$$

eq (15) & (16) are also called as D Alembertz equations

$\frac{1}{\mu\varepsilon} = v$.

velocity (v) depends on the parameters μ & ε

---

① In a material for which conductivity $\sigma = 4.5$ v/m
in a material for which conductor is $\varepsilon_r = 1$.
and the electric field $E = 300 \sin 10^{9} t$ v/m. Determine
the conduction & displacement current densities
and also find the frequency at which they have
equal magnitudes.

Soln:-

given, $\sigma = 4.5$ v/m

$\varepsilon_r = 1$

$E = 300 \sin 10^{9} t$ v/m.

$J_c = \sigma \times E$

$= 4.5 \times 300 \sin 10^{9} t$.

$J_c = 1350 \sin 10^{9} t$ A/m^2 //

$\frac{\partial D}{\partial t} = \varepsilon \frac{\partial}{\partial t} E$

$= \varepsilon \frac{\partial}{\partial t} (300 \sin 10^{9} t)$

$= \varepsilon_0 \varepsilon_r \frac{\partial}{\partial t} (300 \sin 10^{9} t)$.

$$\frac{\partial D}{\partial t} = 8.854 \times 10^{-12} \frac{\partial}{\partial t} (300 \sin 10^9 t)$$

$$= 8.854 \times 10^{-12} \cos 10^9 t \, (10^9)(300)$$

$$= 8.854 \times 10^{-3} \cos 10^9 t \, (300)$$

$$= 2.656 \cos 10^9 t \qquad = 2.652 \times 10^{-10} \ A/m^2$$

28/05/23

3rd period.

Wave Equations And Their Solutions

Maxwell's second eqn in pointed form

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad -① .$$

Take curl on both sides of eq ①

$$\nabla \times \nabla \times E = -\mu \left[\nabla \times \frac{\partial H}{\partial t} \right] \quad -②$$

Maxwell's first eqn in point form

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t} \quad -③$$

differentiating eq (3)

$$\nabla \times \frac{\partial H}{\partial t} = \sigma \frac{\partial E}{\partial t} + \frac{\partial^2 D}{\partial t^2} \quad \text{— (4)}$$

sub eq (4) in (2)

$$\nabla \times \nabla \times E = -\mu \left[\sigma \frac{\partial E}{\partial t} + \frac{\partial^2 D}{\partial t^2} \right] \quad \text{— (5)}$$

Using an Identity,

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\nabla \times E = \frac{\nabla \cdot D}{\varepsilon}$$

$$= \frac{1}{\varepsilon}\left[\nabla \cdot D\right] \quad \text{— (6)}$$

Material is taken as a conductor, $(\rho = 0)$. → when a conductor

$$\nabla \cdot D = \rho$$

$$\nabla \cdot D = 0 \,//$$

$$\therefore \quad \nabla \cdot E = 0 \,//$$

Sub in identity,

$$\nabla \times \nabla \times E = -\nabla^2 E \quad \text{— (7)}$$

Compare ex (5) & (6)
equate

$$\nabla \times \nabla \times E = -\nabla^2 E$$

$$-\nabla^2 E = -\left[\mu\sigma \frac{\partial E}{\partial t} + \mu \frac{\partial^2 D}{\partial t^2} \right]$$

$$\nabla^2 \mathcal{E} - \mu\sigma \frac{\partial \mathcal{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \mathcal{E}}{\partial t^2} = 0 \quad \text{--- (8)}$$

Ean (8) is the wave equation for electric field.

—

Define wave ean in terms of mag. field.

Maxwell's first ean in point form.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times H = \sigma \mathcal{E} + \varepsilon \frac{\partial \mathcal{E}}{\partial t} \quad \text{--- (9)}$$

Diff eq (9)

Take curl on both sides,

$$\nabla \times \nabla \times H = \sigma \nabla \mathcal{E} + \varepsilon \nabla \times \frac{\partial \mathcal{E}}{\partial t}$$

Maxwell's (2) ean in point form.

$$\nabla \times \mathcal{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \mathcal{E} = -\mu \frac{\partial H}{\partial t} \quad \text{--- (10)}$$

Diff (10) wrt time

$$\nabla \times \frac{\partial \mathcal{E}}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2} \quad \text{--- (11)}$$

sub (1) in (10)

$$\cancel{\nabla \times \frac{\partial \varepsilon}{\partial t}} = \cancel{[\nabla \times \varepsilon]\varepsilon}$$

$$\nabla \times \nabla \times H = \sigma \nabla \varepsilon + \varepsilon \left[-\mu \frac{\partial^2 H}{\partial t^2} \right] \quad \text{— (12)}$$

Using an vector identity,

$$\nabla \times \nabla \times H = \nabla (\nabla \cdot H) - \nabla^2 H$$

$$\nabla \times \nabla \times H = 0 \text{ //}$$

$$\nabla \times H = \frac{\nabla \cdot B}{\mu_0} \qquad \nabla \times H = (\sigma + j\omega\varepsilon) E$$

$$\nabla \times H = \frac{1}{\mu} [\nabla \cdot B] \quad \text{— (13)}$$

within a conductor,

$$\nabla \cdot B = 0$$

$$\nabla \cdot \mu H = 0$$

$$\nabla \cdot H = 0 \text{ //}$$

sub $\nabla \cdot H$

$$\nabla \times \nabla \times H = \nabla (0) - \nabla^2 H$$

$$\nabla \times \nabla \times H = -\nabla^2 H \text{ //}$$

Sub,

$$-\nabla^2 H = \sigma\left(-\mu \frac{\partial H}{\partial t}\right) + \varepsilon\left[-\mu \frac{\partial^2 H}{\partial t^2}\right]$$

$$+\nabla^2 H = \sigma \mu \frac{\partial H}{\partial t} + \varepsilon\mu\left[\frac{\partial^2 H}{\partial t^2}\right]$$

$$\nabla^2 H - \sigma\mu\left[\frac{\partial H}{\partial t}\right] - \varepsilon\mu\left[\frac{\partial^2 H}{\partial t^2}\right] = 0$$

wave equation in terms of mag. field.

25/5/23

3rd period.

$$\gamma = \alpha + j\beta$$

(i.e) Propagation constant = ~~Actual~~ Attenuation constant & Phase constant

case i)

$\gamma \rightarrow$ real $[\gamma = \alpha]$, $\beta = 0$.

$\downarrow$

No wave propagation

Case ii)

$\gamma \rightarrow$ imaginary $(\gamma = j\beta)$, $\alpha = 0$.

* In high frequencies used.

Wave equation in Phasor form :-

Maxwell equation in Phasor form :-

Maxwell's first equation,

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\left[\because \quad \frac{\partial D}{\partial t} = j\omega D\right]$$

$$\nabla \times H = \sigma E + j\omega \varepsilon E$$

$$\boxed{\nabla \times H = (\sigma + j\omega \varepsilon)\, E}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -j\omega \mu H}$$

wave eqn in E

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 E - \mu\sigma (j\omega) - \mu\varepsilon (j\omega)^2 E = 0$$

$$\nabla^2 E - \underbrace{j\omega\mu (\sigma + j\omega \varepsilon E)}_{\gamma^2} = 0$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\Rightarrow \nabla^2 E = \gamma^2 E$$

Illy

$$\nabla^2 H - \gamma^2 H = 0$$

$$\Rightarrow \nabla^2 H = \gamma^2 H$$

$$\gamma^2 = j\omega\mu\left[\sigma + j\omega\varepsilon_0\varepsilon\right]$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon_0\varepsilon)}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \sqrt{\frac{\mu\varepsilon}{2}\left(\left(\sqrt{\left(1+\frac{\sigma}{\omega\varepsilon}\right)^2}\right)^2 - 1\right)} \quad \text{nepee / metre}$$

$$\beta = \sqrt{\frac{\mu\varepsilon}{2}\left[\left(\sqrt{\left(1+\frac{\sigma}{\omega\varepsilon}\right)^2}\right)^2 - 1\right]} \quad \text{radius / metre}$$

Intrinsic impedence:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \quad \text{ohms}$$

Ratio b/w electric field and magnetic field.

Free space $\sigma = 0$

$$\eta_0 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon_0}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \text{ohms} \qquad \eta_0 = 120\pi \text{ (or) } \simeq 377\,\Omega$$

wavelength (λ)

$$\lambda = \frac{v}{f} = \frac{2\pi}{\beta}$$

Free space $v = c$

$$\lambda = \frac{c}{f}$$

1) A 6580 MHz uniform plane wave is propagating in a material medium of $\varepsilon_r = 2.25$. If the amplitude of electric field intensity of loss less medium is 500 volts/m. Calculate

$\alpha = 0$

i) Phase constant β

ii) Propagation constant

iii) Phase velocity

iv) wavelength

v) Intrinsic impedence

vi) Amplitude of mag. field intensity.

Soln:-

i) $\beta = \omega\sqrt{\mu\varepsilon}$

$= 2\pi f \sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}$

$= 2 \times 3.14 \times 6580 \times 10^6 \sqrt{4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 2.25}$

$= 206.859$ ~~rad/m~~ //

ii)

$\gamma = \alpha + j\beta$

$= 0 + j(206.859)$ //

iii) $v_p = \frac{\omega}{\beta}$

$= \frac{2 \times 3.14 \times 6580 \times 10^6}{206.859}$

$\simeq 200 \times 10^6$ m/s.//

iv)

$\lambda = \frac{2\pi}{\beta}$

$= \frac{2 \times 3.14}{206.859}$

$= 0.030$

v)

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$.

$\eta = \sqrt{\frac{\mu}{\varepsilon_0}}$

$= \sqrt{\frac{4\pi \times 10^{-7} \times 1}{8.854 \times 10^{-12} \times 2.25}}$

$=$

vi)

$\varepsilon = 500$ v/m.

$\eta = \frac{\varepsilon}{h}$

$= \frac{E}{h}$

$= \frac{500}{251.156}$

$= 1.99$ Ampere/meter.

Plane wave ~~in~~ conductor

$\frac{\sigma}{\omega\varepsilon} >> 1$, $\sigma >> \omega\varepsilon$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}$$

[Note: $\beta = \sqrt{\ }$ e and -1 final ans will be same by elemenating 1]

$$= \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\frac{\sigma}{\omega\varepsilon}\right]}$$

$$= \omega \sqrt{\frac{\mu\varepsilon}{2} \times \frac{\sigma}{\omega\varepsilon}}$$

$$= \sqrt{\frac{\omega^2 \mu \sigma}{2\omega}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2\pi f \mu\sigma}{2}}$$

$$\alpha = \sqrt{\pi f \mu \sigma}$$ //

$$\boxed{\therefore \alpha = \sqrt{\pi f \mu \sigma} \text{ nepers/m}}$$ //

$||^{ly}$,

$$\therefore \beta = \sqrt{\pi f \mu\sigma} \text{ radians/m}$$ //

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$\sigma >> \omega\varepsilon$, $\sigma + j\omega\varepsilon$ will be neglected. Because conductor value will be high.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \ \Omega}$$

$\left(\frac{E}{H}\right)$

↳ E leads H by 45°

Plane wave in conductor

$$V = \frac{\omega}{\beta} = \frac{2\pi f}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{\sqrt{2\pi} \times \sqrt{2\pi} \times \sqrt{f} \times \sqrt{f}}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{2\pi\sqrt{f}}{\sqrt{\mu\sigma}}$$

$$V = \sqrt{\frac{2\omega}{\mu\sigma}}$$

Skin depth (or) depth of penetration (δ)

The E or H of the wave travels in a conducting medium it's amplitude is attenuated by $e^{-\alpha z}$. The distance through which the wave amplitude is attenuated by the factor $e^{-\alpha z}$.

The distance through which the wave amplitude decreases by the factor e^{-1} (about 37% of the original value) is called skin depth or penetration depth of the medium.

$$\delta = \frac{1}{\alpha}$$

Hint: skin depth is the measure of depth to which an electromagnetic wave can penetrate the medium.

① A uniform plane wave in a medium having conductivity $\sigma = 10^{-3}$ S/m. $\varepsilon = 80\varepsilon_0$, $\mu = \mu_0$ is having a frequency of 10 kHz.

i) verify the medium is a good conductor or not.

ii) calculate $\alpha, \beta, \gamma, \eta, \lambda, v$.

soln:-

$$\frac{\sigma}{\omega\varepsilon} >> 1$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-3}}{2\pi f(80\varepsilon_0)} = \frac{10^{-3}}{2\pi(10)\times10^{3}\times8.854\times10^{-12}}$$

=

$$\alpha = \sqrt{\pi f \mu \sigma}$$

$$= \sqrt{\pi \times 10 \times 10^3 \times \mu_0 \times 10^{-3}}$$

$$= \sqrt{\pi \times 10 \times 4\pi \times 10^{-10} \times 10^3}$$

$$= \sqrt{40\pi^2 \times 10^{-7}}$$

$$\therefore \alpha = 6.28 \times 10^{-3}$$

$$\beta = 6.28 \times 10^{-3}$$

$$\gamma = \alpha + j\beta$$

$$= 6.28 \times 10^{-3} + j(6.28 \times 10^{-3})$$

$$= 8.88 \times 10^{-3} \angle 45'$$

Polar form $\gamma\angle\theta$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45'$$

$$= \sqrt{\frac{2\pi \times 10^3 \times 4\pi \times 10^{-7} \times 10^{-3}}{10^3}} \angle 45'$$

$$= \sqrt{}$$