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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT211 – ELECTROMAGNETIC FIELDS

II YEAR/ IV SEMESTER

UNIT 4 – TIME VARYING FIELDS & MAXWELL'S EQUATION

TOPIC 4 – SCALAR AND VECTOR MAGNETIC POTENTIAL

SCALAR AND VECTOR MAGNETIC POTENTIAL

In electrostatics, it is seen that there exists a scalar electric potential V which is related to the electric field intensity $\bar{E}$ as $\bar{E} = -\nabla V$.

Is there any scalar potential in magnetostatics related to magnetic field intensity $\bar{H}$?

In case of magnetic fields there are two types of potentials which can be defined :

1. The scalar magnetic potential denoted as V_m.
2. The vector magnetic potential denoted as $\bar{A}$.

To define scalar and vector magnetic potentials, let us use two vector identities which are listed as the properties of curl, earlier.

$$\nabla \times \nabla V = 0, \qquad V = \text{Scalar} \qquad \ldots (1)$$

$$\nabla \cdot (\nabla \times \bar{A}) = 0, \qquad \bar{A} = \text{Vector} \qquad \ldots (2)$$

Every Scalar V and Vector $\bar{A}$ must satisfy these identities.

SCALAR AND VECTOR MAGNETIC POTENTIAL

If V_m is the scalar magnetic potential then it must satisfy the equation (1),

$$\therefore \qquad \nabla \times \nabla V_m = 0 \qquad \text{... (3)}$$

But the scalar magnetic potential is related to the magnetic field intensity $\bar{H}$ as,

$$\bar{H} = -\nabla V_m \qquad \text{... (4)}$$

Using in equation (3),

$$\therefore \qquad \nabla \times (-\bar{H}) = 0 \qquad \text{i.e.} \quad \nabla \times \bar{H} = 0 \qquad \text{... (5)}$$

$$\text{But} \qquad \nabla \times \bar{H} = \bar{J} \qquad \text{i.e.} \quad \bar{J} = 0 \qquad \text{... (6)}$$

SCALAR AND VECTOR MAGNETIC POTENTIAL

Thus scalar magnetic potential V_m can be defined for source free region where $\bar{J}$ i.e. current density is zero.

$$\therefore \quad \bar{H} = -\nabla V_m \quad \text{only for } \bar{J} = 0 \quad \ldots (7)$$

Similar to the relation between $\bar{E}$ and electric scalar potential, magnetic scalar potential can be expressed interms of $\bar{H}$ as,

$$V_{m\,a,b} = -\int_{b}^{a} \bar{H} \cdot d\bar{L} \quad \ldots \text{specified path}$$

LAPLACE'S EQUATION FOR SCALAR MAGNETIC POTENTIAL

It is known that as monopole of magnetic field is non existing,

$$\oint \bar{B} \cdot d\bar{S} = 0 \quad \text{... (8)}$$

Using Divergence theorem,

$$\oint \bar{B} \cdot d\bar{S} = \int_{vol} (\nabla \cdot \bar{B})\, dv = 0 \quad \text{... (9)}$$

$$\therefore \quad \nabla \cdot \bar{B} = 0 \quad \text{... (10)}$$

$$\therefore \quad \nabla \cdot (\mu_0 \bar{H}) = 0 \qquad \text{but } \mu_0 \neq 0 \quad \text{... (11)}$$

$$\therefore \quad \nabla \cdot \bar{H} = 0 \quad \text{... (12)}$$

$$\therefore \quad \nabla \cdot (-\nabla V_m) = 0 \qquad \text{... using } \bar{H} = -\nabla V_m$$

$$\therefore \quad \boxed{\nabla^2 V_m = 0 \qquad \text{for} \quad \bar{J} = 0} \quad \text{... (13)}$$

This is Laplace's equation for scalar magnetic potential. This is similar to the Laplace's equation for scalar electric potential $\nabla^2 V = 0$.

VECTOR MAGNETIC POTENTIAL

The vector magnetic potential is denoted as $\bar{A}$ and measured in Wb/m. It has to satisfy equation (2) that divergence of a curl of a vector is always zero.

$$\therefore \quad \nabla \cdot (\nabla \times \bar{A}) = 0 \qquad \text{... } \bar{A} = \text{Vector magnetic potential}$$

$$\text{But} \quad \nabla \cdot \bar{B} = 0 \qquad \text{... From equation (10)}$$

$$\therefore \quad \boxed{\bar{B} = \nabla \times \bar{A}} \qquad \text{... (14)}$$

Thus curl of vector magnetic potential is the flux density.

$$\text{Now} \quad \nabla \times \bar{H} = \bar{J}$$

$$\therefore \quad \nabla \times \frac{\bar{B}}{\mu_0} = \bar{J} \qquad \text{... } \bar{B} = \mu_0 \bar{H}$$

VECTOR MAGNETIC POTENTIAL

$$\therefore \qquad \nabla \times \bar{B} = \mu_0 \bar{J} \qquad \ldots \bar{B} = \nabla \times \bar{A}$$

$$\therefore \qquad \nabla \times \nabla \times A = \mu_0 J \qquad \ldots (15)$$

Using vector identity to express left hand side we can write,

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$$\therefore \qquad \bar{J} = \frac{1}{\mu_0}\left[\nabla \times \nabla \times \bar{A}\right] = \frac{1}{\mu_0}\left[\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}\right] \qquad \ldots (16)$$

Thus if vector magnetic potential is known then current density $\bar{J}$ can be obtained. For defining $\bar{A}$ the current density need not be zero.

POISSON'S EQUATION FOR MAGNETIC FIELD

In a vector algebra, a vector can be fully defined if its curl and divergence are defined.

For a vector magnetic potential A, its curl is defined as $\nabla \times \bar{A} = \bar{B}$ which is known.

But to completely define $\bar{A}$ its divergence must be known. Assume that $\nabla \cdot \bar{A}$, the divergence of $\bar{A}$ is zero. This is consistent with some other conditions to be studied later in time varying magnetic fields. Using in equation (16),

$$\bar{J} = \frac{1}{\mu_0}\left[-\nabla^2 \bar{A}\right]$$

$$\therefore \quad \boxed{\nabla^2 \bar{A} = -\mu_0 \bar{J}} \quad \ldots (17)$$

This is the Poisson's equation for magnetostatic fields.

VECTOR MAGNETIC POTENTIAL DUE TO A DIFFERENTIAL CURRENT ELEMENT

Consider the differential element $d\bar{L}$ carrying current I. Then according to Biot-Savart law the vector magnetic potential $\bar{A}$ at a distance R from the differential current element is given by,

$$\bar{A} = \oint \frac{\mu_0 I \, d\bar{L}}{4\pi R} \text{ Wb/m} \quad \text{... (18)}$$

For the distributed current sources, $Id\bar{L}$ can be replaced by $K\,dS$ where $\bar{K}$ is surface current density.

$$\therefore \quad \bar{A} = \oint_S \frac{\mu_0 \bar{K} \, dS}{4\pi R} \text{ Wb/m} \quad \text{... (19)}$$

The line integral becomes a surface integral. If the volume current density $\bar{J}$ is given in A/m^2 then $I\,d\bar{L}$ can be replaced by $\bar{J}\,dv$ where dv is differential volume element.

$$\therefore \quad A = \oint_{vol} \frac{\mu_0 \bar{J} \, dv}{4\pi R} \text{ Wb/m} \quad \text{... (20)}$$

VECTOR MAGNETIC POTENTIAL DUE TO A DIFFERENTIAL CURRENT ELEMENT

It can be noted that,

1. The zero reference for $\overline{A}$ is at infinity.

2. No finite current can produce the contributions as $R \rightarrow \infty$.

REFERENCES

- John.D.Kraus , “ Electromagnetics “,5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: “Engineering Electromagnetics” Tata McGraw-Hill, 7th Edition 2007

THANK YOU