Module 4 : Uniform Plane Wave

Lecture 31 : Power Flow in a Medium

Objectives

In this course you will learn the following

- The Poynting theorem and the concept of Poynting vector.
- Average Poynting vector(A true measure of power flow).
- Conditions for average power flow.
- Power flow density of a uniform plane wave in different media.
- Concept of surface current and surface impedance.
- Power loss in a conducting surface in terms of linear surface current density and surface resistance.

Power Flow and Poynting Vector

- In general, it is important to know how much power flow is associated with time varying electric and magnetic fields.
- For a uniform plane wave, it is easy to see that the power flows in the direction of the wave. However, if the field distribution is complicated it is not very easy to visualize the direction and quantity of the power flow.
- The concepts of Poynting Vector is very useful in these situations. According to Poynting theorem

$$\text{Net Outward Power from a closed surface } W = \oint_S (\mathbf{E} \times \mathbf{H}) \,.\, da$$

- The quantity $(\mathbf{E} \times \mathbf{H})$ therefore represents the power density on the surface of the closed surface. The vector defined as

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

is called the Poynting vector. The poynting vector gives the power flow per unit area at any location.

- It is important to see that the power flows in a direction perpendicular to both $\mathbf{E} \,\&\, \mathbf{H}$

Instantaneous and Average Poynting Vector

- For $\mathbf{E}$ and $\mathbf{H}$ which are varying sinusoidally, it is rather useful to have the average power density. Writing $\mathbf{E}$ and $\mathbf{H}$ explicitly for the time harmonic function we have

$$\mathbf{E}(x,y,z,t) = \mathbf{E_0}(x,y,z)e^{j\omega t}$$

and

$$\mathbf{H}(x,y,z,t) = \mathbf{H_0}(x,y,z)e^{j\omega t}$$

- The average Poynting vector(average over the time period) is

$$\mathbf{P_{av}} = \frac{1}{2} Re\left(\mathbf{E} \times \mathbf{H}^*\right)$$

- The average power density is a much meaningful quantity as it gives the actual power flow at that location. The instantaneous power on the other hand does not correctly represent the power flow as it can be negative or positive. It can probably then give the amount of power oscillating back and forth around a point plus the actual power flow at that point.
- For a real power flow, two conditions should be satisfied.

(1) $\mathbf{E}$ and $\mathbf{H}$ fields should cross each other

(2) $\mathbf{E}$ and $\mathbf{H}$ should not be in time quardrature i.e., 90^o out of phase with each other. $\mathbf{E}$ and $\mathbf{H}$ fields which are parallel and/or in time quardrature, do not constitute any power flow.

Power Density of a Uniform Plane Wave

- Let us consider a uniform plane wave travelling in $+z$ direction and having its electric field oriented along the $+x$ direction and magnetic field oriented in $+y$ direction.

$$\mathbf{E} = E_0 e^{j\omega t}\hat{\mathbf{x}}$$
$$\mathbf{H} = H_0 e^{j\omega t}\hat{\mathbf{y}} = \frac{E_0}{\eta} e^{j\omega t}\hat{\mathbf{y}}$$

- The average power density of the wave therefore is

$$\begin{aligned}
\mathbf{P_{av}} &= \frac{1}{2}Re\{\mathbf{E}\times\mathbf{H}^*\} \\
&= \frac{1}{2}Re\{E_0 e^{j\omega t}[\frac{E_0}{\eta}e^{j\omega t}]^*\}\hat{\mathbf{x}}\times\hat{\mathbf{y}} \\
&= \frac{1}{2}Re\{\frac{|E_0|^2}{\eta^*}\}\hat{\mathbf{z}} = \frac{1}{2}Re\{\eta|H_0|^2\}\hat{\mathbf{z}} \\
\Rightarrow \quad P_{av} &= \frac{|E_0|^2}{2}Re\{\frac{1}{\eta^*}\} = \frac{|H_0|^2}{2}Re\{\eta\}
\end{aligned}$$

- For a **loss-less dielectric medium** $\eta = \sqrt{\frac{\mu}{\epsilon}} = Real\ number$

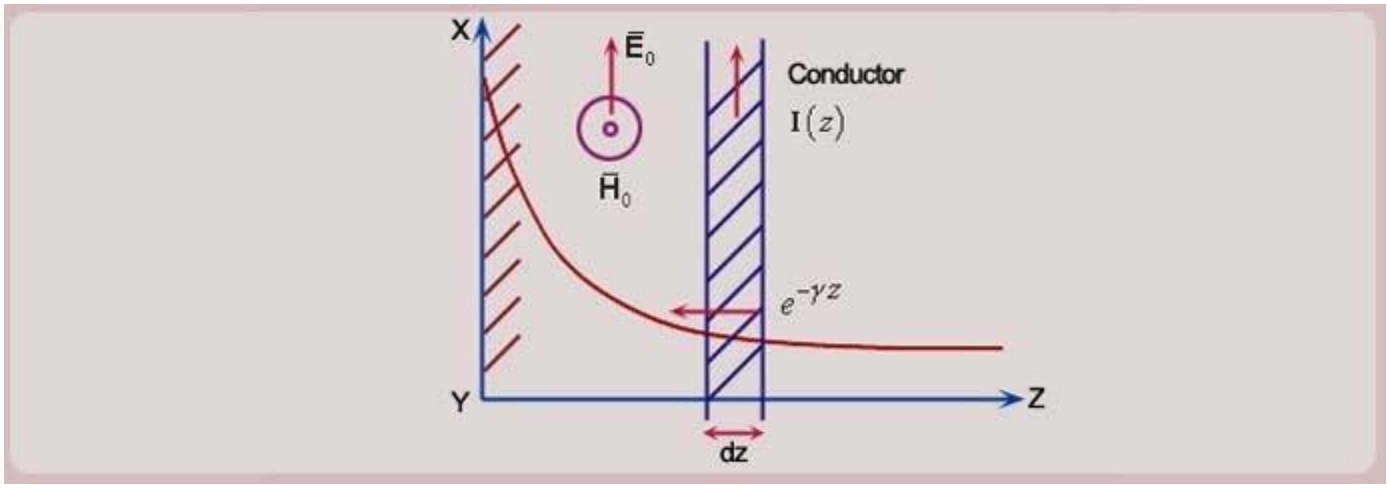
  The average power density of the wave is

$$P_{av} = \frac{1}{2}\frac{|E_0|^2}{\eta} = \frac{1}{2}|E_0|^2\sqrt{\frac{\epsilon}{\mu}}$$

- For a **good conductor** $\sigma >> \omega\epsilon$, and $\eta \approx \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$. The phase angle between $\mathbf{E}$ and $\mathbf{H}$ is 45^o, and the average power density is

Surface Current

- Let the electric field be $\mathbf{E}_0$ at the surface of the conductor



- For a good conductor the propogation constant of the wave is

$$\gamma = \alpha + j\beta = \sqrt{\frac{\omega\mu_0\sigma}{2}} + j\sqrt{\frac{\omega\mu_0\sigma}{2}}$$

- The field at any distance z inside the conductor is given as

$$\mathbf{E}(z) = \mathbf{E_0}e^{-\gamma z} = \mathbf{E_0}e^{-\alpha z}.e^{-j\beta z}$$

- The field amplitude therefore decreases exponentially inside the conductor.

- The conduction current density at a depth z is

$$\mathbf{J}(z) = \sigma\mathbf{E} = \sigma E_0 e^{-\alpha z}e^{-j\beta z}\hat{\mathbf{x}}$$

- The current inside the sheet shown by the hatched region is

$$\mathbf{I}(z) = \mathbf{J}(z)dz = \sigma E_0 e^{-\gamma z}dz\hat{\mathbf{x}}$$

- The total current under the unit width of the conductor surface is therefore

$$\mathbf{J}_s = \int_0^\infty \mathbf{E}_0 \sigma e^{-\gamma z}dz = \mathbf{E}_0\sigma\left[-\frac{e^{-\gamma z}}{\gamma}\right]_0^\infty$$

$$= \frac{E_0\sigma}{\gamma}\hat{\mathbf{x}}$$

- Since for a good conductor the current is confined to a very thin region below the surface, we may treat the current $\mathbf{J}_s$ as the surface current.

- **The true surface current only exist when the conductivity is infinite**

  Note

- For non-ideal conductor a parameter called surface impedance is defined as

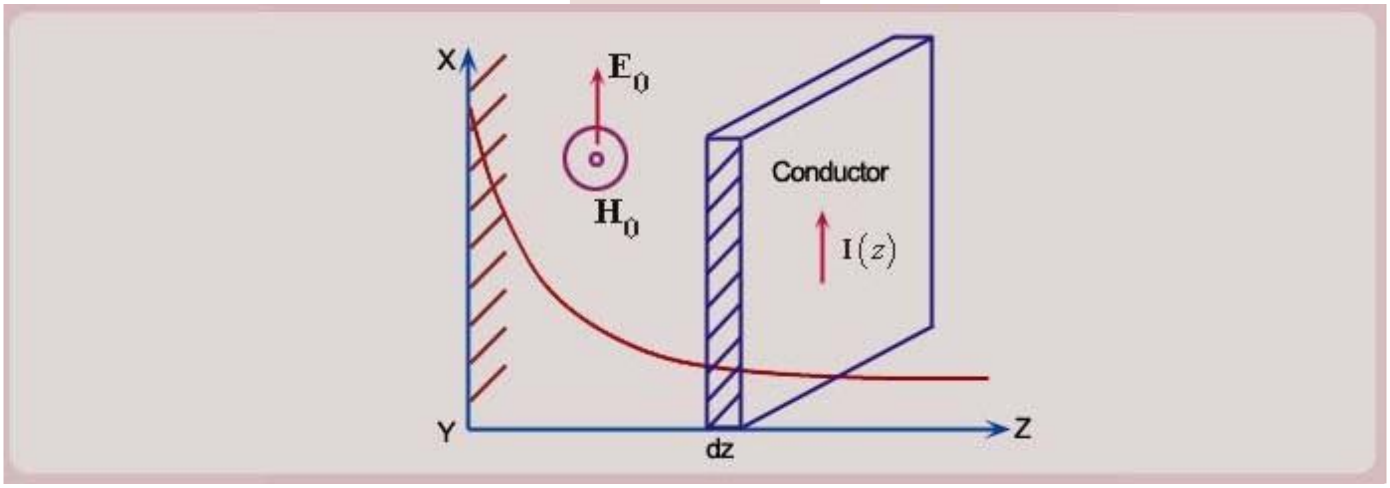
$$Z_s = \frac{|\mathbf{E}_{\tan}|}{|\mathbf{J}_s|} = \frac{E_0}{J_s} = \frac{\gamma}{\sigma} = \eta_c$$
$$= R_s + jX_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} + j\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

- The real part of $\mathbf{Z}_s$ is called the surface resistance and its value is $\sqrt{\frac{\omega\mu_0}{2\sigma}}$

Power Loss in a Conductor

- The resistance of the slab along the direction of the current I is

$$dR = \frac{\rho l}{A} = \frac{1}{\sigma dz}$$



- The ohmic loss in the slab is

$$dW = |I(z)|^2 dR$$

- Substituting for I(z) we get

$$\begin{aligned} dW &= |\sigma E_0 e^{-\gamma z} dz|^2 \frac{1}{\sigma dz} \\ &= \sigma |E_0|^2 e^{-2\alpha z} dz \end{aligned}$$

- The total loss per unit area of the conductor surface therefore is

$$\begin{aligned} W &= \int_0^\infty \sigma |E_0|^2 e^{-2\alpha z} dz \\ &= \sigma |E_0|^2 \left[\frac{e^{-2\alpha z}}{-2\alpha} \right]_0^\infty \\ \Rightarrow \quad W &= \frac{\sigma |E_0|^2}{2\alpha} = \frac{\sigma}{2\alpha} \frac{|\gamma|^2}{\sigma^2} |\mathbf{J}_s|^2 \end{aligned}$$

- Substituting for γ and α , the loss per unit area of the conducting surface is

$$W = R_s |\mathbf{J}_s|^2$$

- The power loss is proportional to the surface resistance which increases with frequency and decreases with conductivity. Higher the conductivity lesser the loss and for ideal conductor when the conductivity is infinite the ohmic loss is zero.

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Recap

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