SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT211 – ELECTROMAGNETIC FIELDS

II YEAR/ IV SEMESTER

UNIT 4 – TIME VARYING FIELDS & MAXWELL'S EQUATION

TOPIC 1 – FARADAY'S LAW-, DISPLACEMENT CURRENT

INTRODUCTION – TIME VARYING FIELDS

- In the previous chapters, we have studied the basic concepts in an electrostatic and magnetostatic fields.
- The fields are considered as time invariant or static fields
- But in time varying or dynamic EM fields, the electric and magnetic fields are interdependent
- Static electric fields are produced by stationary electric charges
- The magnetic fields are produced due to the motion of electric charges with uniform velocity or the magnetic charges
- The time varying fields are produced due to time varying currents

FARADAY'S LAW

- 1791 – 1867
- British physicist and chemist
- Great experimental scientist
- Contributions to early electricity include:
  - Invention of motor, generator, and transformer
  - Electromagnetic induction
  - Laws of electrolysis

FARADAY'S LAW

In year 1820, Prof. Hans Christian Oersted demonstrated that a compass needle deflected due to an electric current. After ten years, Michael Faraday, a British Scientist, proved that a magnetic field could produce a current.

According to Faraday's experiment, a static magnetic field cannot produce any current flow. But with a time varying field, an electromotive force (e.m.f.) induces which may drive a current in a closed path or circuit. This e.m.f. is nothing but a voltage that induces from changing magnetic fields or motion of the conductors in a magnetic field. Faraday discovered that the induced e.m.f. is equal to the time rate of change of magnetic flux linking with the closed circuit.

Faraday's law can be stated as,

$$e = -N\frac{d\phi}{dt} \text{ volts.} \quad \dots (1)$$

where N = Number of turns in the circuit

e = Induced e.m.f.

FARADAY'S LAW

Let us assume single turn circuit i.e. N = 1, then Faraday's law can be stated as,

$$e = -\frac{d\phi}{dt} \text{ volts} \quad \ldots (2)$$

The minus sign in equations (1) and (2) indicates that the direction of the induced e.m.f. is such that to produce a current which will produce a magnetic field which will oppose the original field.

In 1834, **Henri Frederic Emile Lenz** postulated the law. Thus according to **Lenz's law,** the induced e.m.f. acts to produce an opposing flux.

FARADAY'S LAW

Let us consider Faraday's law. The induced e.m.f. is a scalar quantity measured in volts. Thus the induced e.m.f. is given by,

$$e = \oint \bar{E} \cdot d\bar{L} \qquad \dots (3)$$

The induced e.m.f. in equation (3) indicates a voltage about a closed path such that if any part of the path is changed, the e.m.f. will also change.

The magnetic flux ϕ passing through a specified area is given by,

$$\phi = \int_S \bar{B} \cdot d\bar{S}$$

where $\quad B$ = Magnetic flux density

FARADAY'S LAW

Using above result, equation (2) can be rewritten as,

$$e = -\frac{d}{dt}\int_S \bar{B} \cdot d\bar{S}$$

From equations (3) and (4), we get,

$$\boxed{e = \oint \bar{E} \cdot d\bar{L} = -\frac{d}{dt}\int_S \bar{B} \cdot d\bar{S}}$$

FARADAY'S LAW

There are two conditions for the induced e.m.f. as explained below.

i) The closed circuit in which e.m.f. is induced is stationary and the magnetic flux is sinusoidally varying with time. From equation (5) it is clear that the magnetic flux density is the only quantity varying with time. We can use partial derivative to define relationship as $\bar{B}$ may be changing with the co-ordinates as well as time. Hence we can write,

$$\oint \bar{E} \cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \quad \text{... (6)}$$

This is similar to transformer action and e.m.f. is called **transformer e.m.f.** Using Stoke's theorem, a line integral can be converted to the surface integral as

$$\oint_S (\nabla \times \bar{E}) \cdot d\bar{S} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \quad \text{... (7)}$$

Assuming that both the surface integrals taken over identical surfaces.

FARADAY'S LAW

$$\therefore \qquad (\nabla \times \bar{E}) \cdot d\bar{S} = -\frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$

Hence finally,

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}} \qquad \text{... (8)}$$

Equation (8) represents one of the Maxwell's equations. If $\bar{B}$ is not varying with time, then equations (6) and (8) give the results obtained previously in the electrostatics.

$$\oint \bar{E} \cdot d\bar{L} = 0, \text{ and}$$

$$\nabla \times \bar{E} = 0$$

FARADAY'S LAW

$$\therefore \quad (\nabla \times \bar{E}) \cdot d\bar{S} = -\frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$

Hence finally,

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}} \quad \ldots (8)$$

Equation (8) represents one of the Maxwell's equations. If $\bar{B}$ is not varying with time, then equations (6) and (8) give the results obtained previously in the electrostatics.

$$\oint \bar{E} \cdot d\bar{L} = 0, \text{ and}$$

$$\nabla \times \bar{E} = 0$$

FARADAY'S LAW

ii) Secondly magnetic field is stationary, constant not varying with time while the closed circuit is revolved to get the relative motion between them. This action is similar to generator action, hence the induced e.m.f. is called **motional** or **generator e.m.f.**

Consider that a charge Q is moved in a magnetic field $\bar{B}$ at a velocity $\bar{v}$. Then the force on a charge is given by,

$$\bar{F} = Q\,\bar{v} \times \bar{B} \quad \text{... (9)}$$

But the motional electric field intensity is defined as the force per unit charge. It is given by,

$$\therefore \quad \bar{E}_m = \frac{\bar{F}}{Q} = \bar{v} \times \bar{B} \quad \text{... (10)}$$

FARADAY'S LAW

Thus the induced e.m.f. is given by,

$$\oint \bar{E}_m \cdot d\bar{L} = \oint \left(\bar{v} \times \bar{B}\right) \cdot d\bar{L} \quad \text{... (11)}$$

Equation (11) represents total e.m.f. induced when a conductor is moved in a uniform constant magnetic field.

If the directions of velocity $\bar{v}$ with which conductor is moving and the magnetic field $\bar{B}$ are mutually perpendicular to each other, then the induced e.m.f. is given by,

$$e = Blv \sin 90^\circ = Blv \quad \text{..(12)}$$

where l = Length of straight conductor

iii) If in case, the magnetic flux density is also varying with time, then the induced e.m.f. is the combination of transformer e.m.f. and generator e.m.f. given by,

$$\oint \bar{E} \cdot d\bar{L} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} + \oint \left(\bar{v} \times \bar{B}\right) \cdot d\bar{L} \quad \text{... (13}$$

DISPLACEMENT CURRENT

Faraday's experimental law has been used to obtain one of Maxwell's equations in differential form,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{15}$$

which shows us that a time-changing magnetic field produces an electric field.

DISPLACEMENT CURRENT

We should first look at the point form of Ampère's circuital law as it applies to steady magnetic fields,

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{16}$$

and show its inadequacy for time-varying conditions by taking the divergence of each side,

$$\nabla \cdot \nabla \times \mathbf{H} \equiv 0 = \nabla \cdot \mathbf{J}$$

The divergence of the curl is identically zero, so $\nabla \cdot \mathbf{J}$ is also zero. However, the equation of continuity,

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

then shows us that (16) can be true only if $\partial \rho_v / \partial t = 0$. This is an unrealistic limitation, and (16) must be amended before we can accept it for time-varying fields. Suppose we add an unknown term $\mathbf{G}$ to (16),

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{G}$$

DISPLACEMENT CURRENT

Again taking the divergence, we have

$$0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{G}$$

Thus

$$\nabla \cdot \mathbf{G} = \frac{\partial \rho_v}{\partial t}$$

Replacing ρ_v by $\nabla \cdot \mathbf{D}$,

$$\nabla \cdot \mathbf{G} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

from which we obtain the simplest solution for $\mathbf{G}$,

$$\mathbf{G} = \frac{\partial \mathbf{D}}{\partial t}$$

DISPLACEMENT CURRENT

Ampère's circuital law in point form therefore becomes

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (17)$$

The additional term $\partial \mathbf{D}/\partial t$ has the dimensions of current density,

amperes per square meter. Since it results from a time-varying electric flux density (or displacement density), Maxwell termed it a *displacement current density*. We sometimes denote it by $\mathbf{J}_d$:

DISPLACEMENT CURRENT

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d$$

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

This is the third type of current density we have met. Conduction current density,

$$\mathbf{J} = \sigma \mathbf{E}$$

is the motion of charge (usually electrons) in a region of zero net charge density, and convection current density,

$$\mathbf{J} = \rho_v \mathbf{v}$$

is the motion of volume charge density. Both are represented by **J** in (17). Bound current density is, of course, included in **H**. In a nonconducting medium in which no volume charge density is present, $\mathbf{J} = 0$, and then

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (\text{if } \mathbf{J} = 0) \qquad (18)$$

REFERENCES

- John.D.Kraus , “ Electromagnetics “,5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: “Engineering Electromagnetics” Tata McGraw-Hill, 7th Edition 2007

THANK YOU