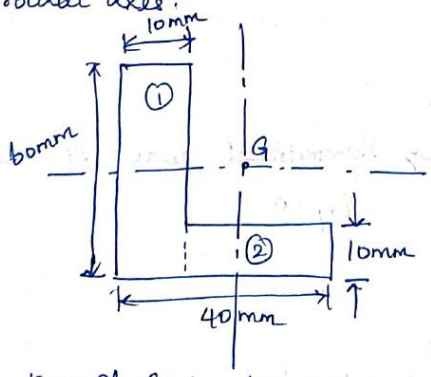
1. Determine the product of inertia of the angle section, shown in figure with respect to its centroidal axes.



Location of Centroid

① (10 x 60 mm)

$a_1 = 10 \times 60 = 600 mm^2$

$x_1 = \frac{10}{2} = 5mm$

$y_1 = \frac{60}{2} = 30mm$

② (30 x 10 mm)

$a_2 = 30 \times 10 = 300 mm^2$

$x_2 = 10 + \left(\frac{30}{2}\right) = 25mm$

$y_2 = \frac{10}{2} = 5mm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{(600 \times 5) + (300 \times 25)}{600 + 300}$$

$$= 11.67 mm$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(600 \times 30) + (300 \times 5)}{600 + 300}$$

$$= 21.67 mm$$

* Product of inertia coordinates

Rectangle ①

$x_1' = -(\bar{x} - x_1)$

$= -[11.67 - 5]$ [∵ centroid is on left of yy axis]

$= -6.67\,mm$

$y_1' = (y_1 - \bar{y})$

$= 30 - 21.67$ [∵ +ve, located above xx axis]

$= 8.33\,mm.$

Rectangle ②

$x_2' = (x_2 - \bar{x}) = 25 - 11.67 = 13.33\,mm$

$y_2' = -(\bar{y} - y_2) = -(21.67 - 5) = -16.67\,mm$

* Product of inertia about centroidal axes.

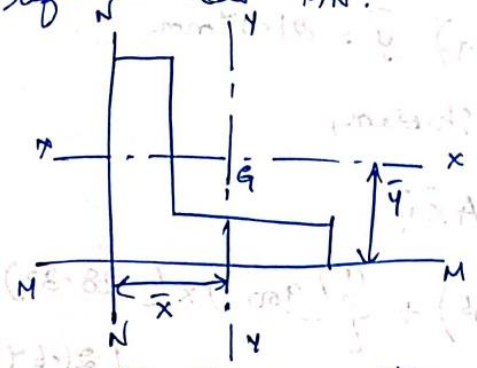
$I_{xy} = \Sigma(I_{xy}) + ax'y'$ | ∵ $(I_{xy})_1 = (I_{xy})_2 = 0$

$= (a_1 x_1' y_1') + (a_2 x_2' y_2')$

$= [\{600 \times (-6.67) \times (8.33)\} + \{(300) \times (13.33) \times (-16.67)\}]$

$= -1 \times 10^5\,mm^4.$

2. In the above problem, determine PoI about the reference axes MN.



We know,

$I_{xy} = -1 \times 10^5 mm^4$

$\bar{x} = 11.67 mm$

$\bar{y} = 21.67 mm$

Both $\bar{x}$ & $\bar{y}$ are positive since axis is right side of NN & above MM axis.

From parallel axis Theorem

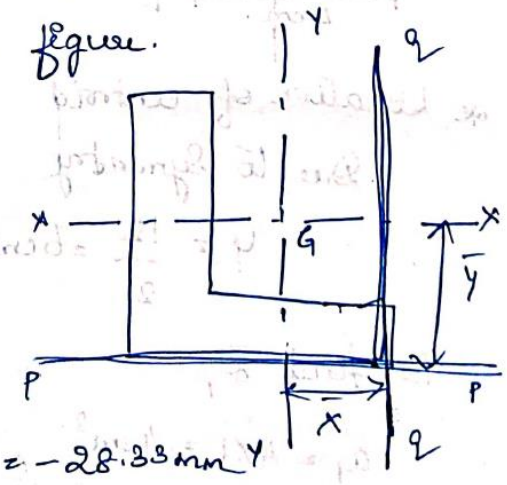
$I_{MN} = I_{xy} + A\bar{x}\bar{y}$

$= (-1 \times 10^5) + \{900 \times 11.67 \times 21.67\}$

$= 1.876 \times 10^5 mm^4.$

$A = 600 + 300$
$= 900 mm^2$

3. In the above problem, determine PoI about pq axes, shown in figure.



We know that

$I_{xy} = -1 \times 10^5$

$\bar{x}$ = Negative

$\bar{y}$ = Positive

$\bar{x} = -(40 - 11.67) = -28.33 mm$

From parallel axis Theorem,

$$I_{pq} = I_{xy} + A\bar{x}\bar{y}$$

$$= (-1 \times 10^5) + \{(900) \times (-28.33) \times (21.64)\}$$

$$= -6.525 \times 10^5 \text{ mm}^4.$$