PRODUCT OF INERTIA & PRINCIPAL MOMENT OF INERTIA

The moment of Inertia of plane figures about a set of perpendicular axes is called product of Inertia. It is required to determine the magnitude & direction of maximum & minimum moment of inertia known as principal moment of Inertia.
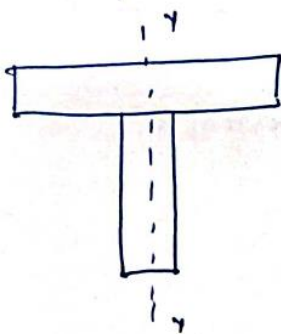
Product of Inertia of whole area $= I_{xy} = \int xy \, dA$.

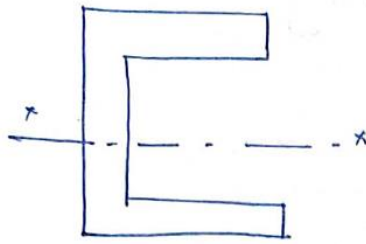Parallel axis Theorem,

$$I_{MN} = I_{xy} = A\bar{x}\bar{y}$$

⇒ Section is Symmetrical about the axes $I_{xy} = 0$.

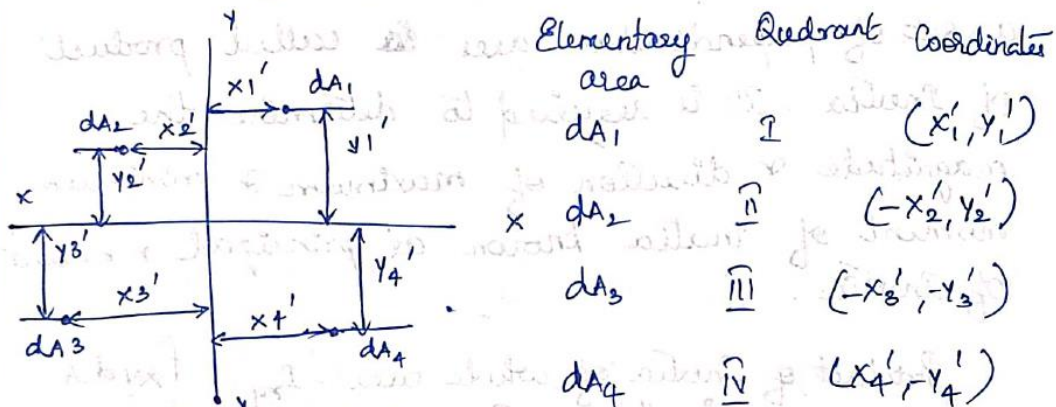⇒ Symmetrical about yy axes
$$I_{xy} = 0$$

Section is Symmetrical about
XX axes
$\therefore I_{xy} = 0.$

POI of plane figure is not ~~Symmetrical~~ determined for the section which is not Symmetrical through sign convention of coordinates of centroid of any elementary area.

Elementary area	Quadrant	Coordinates
dA_1	I	(x_1', y_1')
dA_2	II	$(-x_2', y_2')$
dA_3	III	$(-x_3', -y_3')$
dA_4	IV	$(x_4', -y_4')$

Product of Inertia of Simple figures about Ref axes.
① $I_{MN} = \dfrac{b^2 h^2}{4} (\square)$ ② Right angle $\triangle$ $I_{MN} = h^2 b^2/8$