UNIT-II Vector Calculus

The vector differential operator ∇ is defined as

① $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

② $\nabla\phi = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\phi$

③ $\text{grad}\,\phi = \nabla\phi$

Problems:

1. Find grad ϕ where $\phi = x^2+y^2+z^2$

Sol

$$\phi = x^2+y^2+z^2$$

$$\text{grad}\,\phi = \nabla\phi$$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$= \vec{i}\frac{\partial}{\partial x}(x^2+y^2+z^2) + \vec{j}\frac{\partial}{\partial y}(x^2+y^2+z^2) + \vec{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)$$

$$= \vec{i}(2x) + \vec{j}(2y) + \vec{k}(2z)$$

$$= 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$$

2. Find grad ϕ where $\phi = xyz$ at $(1,1,1)$

Sol

Given $\phi = xyz$

$$\text{grad}\,\phi = \vec{\nabla}\phi$$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$= \vec{i}\frac{\partial}{\partial x}(xyz) + \vec{j}\frac{\partial}{\partial y}(xyz) + \vec{k}\frac{\partial}{\partial z}(xyz)$$

$$= \vec{i}(yz) + \vec{j}(xz) + \vec{k}(xy)$$

$$(\nabla\phi)_{(1,1,1)} = \vec{i}(1.1) + j(1.1) + \vec{k}(1,1)$$

$$\text{grad}\,\phi = \vec{i}+\vec{j}+\vec{k}$$

(3) Find grad ϕ where $\phi = 3x^2y - y^3z^2$ at (1,1)

Given $\phi = 3x^2y - y^3z^2$

grad $\phi = \nabla\phi$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$= \vec{i}\frac{\partial}{\partial x}(3x^2y - y^3z^2) + \vec{j}\frac{\partial}{\partial y}(3x^2y - y^3z^2) + \vec{k}\frac{\partial}{\partial z}(3x^2y - y^3z^2)$$

$$= \vec{i}(6xy) + \vec{j}(3x^2 - 3y^2z^2) + \vec{k}(-2zy^3)$$

$$= 6xy\vec{i} + \vec{j}(3x^2 - 3y^2z^2) - 2y^3z\vec{k}$$

$$= 6(1)(1)\vec{i} + (3 - 3(1)(1))\vec{j} - \vec{k}\,2(1)(1)$$

$$\nabla\phi_{(1,1,1)} = 6\vec{i} + (3-3)\vec{j} - 2\vec{k} = 6\vec{i} + 0\vec{j} - 2\vec{k}$$

(4) Find grad ϕ where $\phi = x^2y^2z^2 + 4xz^2 + xy$ at (1,2,3)

Sol

Given $\phi = x^2y^2z^2 + 4xz^2 + xy$

grad $\phi = \nabla\phi$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$= \vec{i}\frac{\partial}{\partial x}(x^2y^2z^2 + 4xz^2 + xy) + \vec{j}\frac{\partial}{\partial y}(x^2y^2z^2 + 4xz^2 + xy) + \vec{k}\frac{\partial}{\partial z}(x^2y^2z^2 + 4xz^2 + xy)$$

$$= \vec{i}(2xy^2z^2 + 4z^2 + y) + \vec{j}(2yx^2z^2 + x) + \vec{k}(x^2y^2 2z + 8xz)$$

$$\nabla_{(1,2,3)} = \vec{i}$$

$$= \vec{i}$$

$$= 110\vec{i} + 37\vec{j} + 48\vec{k}$$