SOLENOIDAL VECTOR

A vector $\vec{F}$ is said to be solenoidal if $\text{div}.\vec{F} = 0$ or $\nabla.\vec{F} = 0$

Problems:

① Show that the vector $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is solenoidal.

Sol

Given $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$

Solenoidal vector $\nabla.\vec{F} = 0$

$$\nabla\vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\left(3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}\right)$$

$$= \frac{\partial}{\partial x}(3y^4z^2) + \frac{\partial}{\partial y}(4x^3z^2) + \frac{\partial}{\partial z}(-3x^2y^2)$$

$$= 0$$

IRROTATIONAL VECTOR

A vector F is called irrotational if $\text{curl}.\vec{F} = 0$ (or)

$\nabla \times \vec{F} = 0$

Problems:

Show that the vector $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational

Sol

Given $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$

T.P $\nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 6xy+z^3 & 3x^2-z & 3xz^2-y \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(3xz^2 - y) - \frac{\partial}{\partial z}(3x^2 - z)\right] - \vec{j}\left[\frac{\partial}{\partial x}(3xz^2 - y) - \frac{\partial}{\partial z}(6xy + x^3)\right] + \vec{k}\left[\frac{\partial}{\partial x}(3x^2 - z) - \frac{\partial}{\partial y}(6xy + z^3)\right]$$

$$= \vec{i}(-1+1) - \vec{j}(3z^2 - 3z^2) + \vec{k}(6x - 6x)$$

$$= 0$$

2) Find the constants a, b, c ~~so~~ if that the vector is irrotational $\vec{F} = (x+2y+az)\vec{i} + (bx+3y-z)\vec{j} + (4x+cy+2z)\vec{k}$

Sol

Given $\vec{F} = (x+2y+az)\vec{i} + (bx+3y-z)\vec{j} + (4x+cy+2z)\vec{k}$

We know $\nabla \times \vec{F} = 0$

But W.K.T $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+2y+az & bx+3y-z & 4x+cy+2z \end{vmatrix}$$

$$= \vec{i}\left[\partial/\partial y(4x+cy+2z) - \partial/\partial z(bx+3y-z)\right.$$

$$-\vec{j}\left[\frac{\partial}{\partial x}(\right.$$

$$+\vec{k}$$

$$= \vec{i}(c+1) - \vec{j}(4-a) + \vec{k}(b-2)$$

$(c+1) = 0 \qquad (4-a) = 0 \qquad b-2 = 0$

$a = 4, \; b = 2, \; c = -1 \qquad \because \nabla \times F = 0\vec{i} + 0\vec{j} + 0\vec{k}$

ANGLE BETWEEN TWO SURFACE

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

Problems:

① Find the angle between the surface $x = y^2 - 1$ and $x^2 y = 2$ at the point $(1, 1, 1)$

Sol

Given $x = y^2 - 1$; $x^2 y = 2$

$\phi_1 = x - y^2 + 1$, $\phi_2 = x^2 y - 2$

$\nabla \phi_1 = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$

$= \vec{i}$

$= \vec{i} - 2y \vec{j} + 0\vec{k}$

$\nabla \phi_1 (1,1,1) = \vec{i} - 2\vec{j}$

$\nabla \phi_2 = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$

$= \vec{i} \frac{\partial}{\partial x} (x^2 y - 2) + \vec{j} \frac{\partial}{\partial y} (x^2 y - 2) + \vec{k} \frac{\partial}{\partial x} (x^2 y - 2)$

$= \vec{i} (2xy) + \vec{j} (x^2) + \vec{k} (0)$

$\nabla \phi_2 (1,1,1) = \vec{i} (2.1.1) + \vec{j} (1) + \vec{k} (0)$

$= 2\vec{i} + \vec{j}$

$|\nabla \phi_2| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\cos \theta = \dfrac{\nabla \phi_1 \; \nabla \phi_2}{|\nabla \phi_1| \; |\nabla \phi_2|}$

$= \dfrac{(\vec{i} - 2\vec{j}).(2\vec{i} + \vec{j})}{\sqrt{5}.\sqrt{5}}$

$= \dfrac{2 - 2}{5} = 0$

$\cos \theta = 0$

$\theta = \cos^{-1}(0)$

$= \pi/2$