UNIT - I

Multiple Integrals

Application of multiple Integrals are to find areas and volume of various bodies just by taking a little part of them into considerations.

Double Integration (Cartesian coordinates)

A double integration is computed by repeated single variable integration. integrate with respect one variable treating the other variable as constant

ie) $I = \iint f(x,y)\,dx\,dy$, where e is region.

Problems on double integration in cartesian coordinates.

1. Evaluate $\int_0^1 \int_1^2 x(x+y)\,dy\,dx$

Sol

$$\int_0^1 \int_1^2 x(x+y)\,dy\,dx = \int_0^1 \int_1^2 (x^2+xy)\,dy\,dx$$

$$= \int_0^1 \left[x^2 y + x\frac{y^2}{2}\right]_1^2 dx$$

$$= \int_0^1 \left(x^2 \cdot 2 + x\frac{2^2}{2}\right) - \left(x^2 \cdot 1 + x\frac{1}{2}\right) dx$$

$$= \int_0^1 \left(2x^2 + \frac{4x}{2}\right) - \left(x^2 + \frac{x}{2}\right) dx$$

$$= \int_0^1 \left(x^2 + \frac{3x}{2}\right) dx$$

$$= \left(\frac{x^3}{3} + \frac{3x^2}{4}\right)_0^1$$

$$= \frac{1}{3} + \frac{3}{4} = \frac{4+9}{12} = \frac{13}{12}$$

2. Evaluate $\int_0^3 \int_0^2 e^{x+y}\,dy\,dx$

Sol

$$= \int_0^3 \int_0^2 e^x e^y\,dy\,dx$$

$$= \int_0^3 e^x \left(e^y\right)_0^2 dx$$

$$= \int_0^3 e^x (e^2 - e^0)\,dx = (e^2-1)\int_0^3 e^x\,dx$$

3. Evaluate $\int_1^2 \int_1^3 \frac{dx\,dy}{xy}$

Sol

$$\int\int \frac{dx\,dy}{xy} = \int\int \frac{dx}{x}\,\frac{dy}{y}$$

$$= \int [\log x]_1^2 \frac{dy}{y}$$

$$= \int (\log 2 - \log 1)\frac{dy}{y}$$

$$= \int (\log 2/1)\frac{dy}{y}$$

$$= \log 2 [\log y]_2^3$$

$$= \log 2(\log 3 - \log 2)$$

$$= \log 2 (\log 3/2)$$

4. Evalute $\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dx\,dy$

$$= \int\int (x^3 + xy^2)\,dy\,dx$$

$$= \int_0^5 \left[x^3 y + x\frac{y^3}{3}\right]_0^{x^2} dx$$

$$= \int_0^5 \left(x^3 x^2 + \frac{x(x^2)^3}{3}\right) dx$$

$$= \int_0^5 \left(x^5 + \frac{x^7}{3}\right) dx = \left[\frac{x^6}{6} + \frac{x^8}{3\times 8}\right]_0^5$$

$$= \left(\frac{5^6}{6} + \frac{5^8}{24}\right) = 5^6\left(\frac{1}{6} + \frac{5^2}{24}\right)$$

$$= 5^6\left(\frac{4+25}{24}\right) = 5^6\left[\frac{29}{24}\right]$$

H.W

5. Evalute $\int_0^a \int_0^b xy(x-y)\,dx\,dy$

Sol

$$= \int\int (x^2 y - xy^2)\,dx\,dy = \int_0^a \left(\frac{x^3 y}{3} - \frac{x^2 y^2}{2}\right)_0^b dy$$

$$= \int \left(\frac{b^3}{3}y - \frac{b^2 y^2}{2}\right) dx = \left(\frac{b^3 y^2}{b} - \frac{b^2 y^3}{6}\right)_0^a$$

$$= \frac{a^2 b^3}{b} - \frac{a^3 b^2}{6}$$

$$= \frac{a^2 b^2}{b}(b-a)$$