FORCES IN 3D [FORCES IN SPACE)

ment of a Force :

Moment of Force about a point

The moment $\vec{M}$ of a Force $\vec{F}$ with respect to a point O is the cross product $\vec{r} \times \vec{F}$ (Not $\vec{F} \times \vec{r}$) where $\vec{r}$ is the position vector relative to point O, at any point P on the line of action of $\vec{F}$.

* Moment $\vec{M}$ represents the tendency of the Force $\vec{F}$, to rotate a body on which it acts, about an axis called moment axis.

* This moment axis is passing through 'O' & perpendicular to the plane containing the Force $\vec{F}$ & the position vector $\vec{r}$

* The moment vector is usually denoted by a double line arrow $[\vec{M}]$ or by a curved arrow about the moment axis as shown in figure

If $\vec{r}$ & $\vec{F}$ are given,

$$\vec{r} = xi + yj + zk$$

$$\vec{F} = F_x i + F_y j + F_z k$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M} = M_x i + M_y j + M_z k \quad -① $$

$$\vec{M} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (F_z y - F_y z) i + (F_x z - F_z x) j + (F_y x - F_x y) k \quad \text{---} \; (2)$$

Equating (1) & (2) we get,

$$M_x = F_z y - F_y z$$

$$M_y = F_x z - F_z x$$

$$M_z = F_y x - F_x y$$

where, M_x, M_y & M_z are scalar quantities of $\vec{M}$ about x, y & z axes through O.

Magnitude of Moment $\vec{M}$

From the moment vector $\vec{M} = M_x i + M_y j + M_z k$

Magnitude of Moment, $M = \sqrt{M_x^2 + M_y^2 + M_z^2}$

Direction of Moment $\vec{M}$

Let moment vector $\vec{M}$ makes angles ϕ_x, ϕ_y & ϕ_z about x, y & z axes.

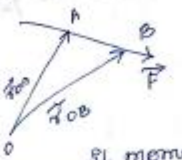
$$\cos\phi = \frac{M_x}{M} \Rightarrow \phi = \cos^{-1}\left(\frac{M_x}{M}\right)$$

|||ly $\phi_y = \cos^{-1}\left(M_y/M\right)$

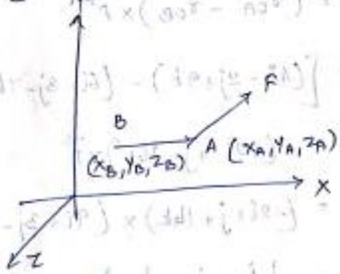
$\phi_z = \cos^{-1}\left(M_z/M\right)$

Note: Point P may be taken any where on the line of action of $\vec{F}$.

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F} = \vec{r}_{OB} \times \vec{F}$$



In case, if moment about any arbitary point B, of force $\vec{F}$ acting at A is required, the relative position vector of A, with respect to B should be used ($r_{A/B}$)



Case (i)

when position vector A & B are known,

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$= (\vec{r}_{OA} - \vec{r}_{OB}) \times \vec{F}$$

Case (ii)

when coordinates of A & B are known

$$\vec{M}_B = \begin{vmatrix} i & j & k \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$