



# **SNS COLLEGE OF TECHNOLOGY**

**Coimbatore-35.**

**An Autonomous Institution**

**COURSE NAME : 19ITB201 DESIGN AND ANALYSIS OF ALGORITHMS**

**II YEAR/ IV SEMESTER**

**UNIT – IV FLOW NETWORKS AND STRING MATCHING**



# The Maximum-Flow Problem

- In this we consider the important problem of maximizing the flow of a material through a transportation network.
- Represented by a connected weighted digraph with  $n$  vertices numbered from 1 to  $n$  and a set of edges  $E$ , with the following properties:
- It contains exactly one vertex with no entering edges; this vertex is called the **source** and assumed to be numbered 1.
- It contains exactly one vertex with no leaving edges; this vertex is called the **sink** and assumed to be numbered  $n$ .
- The weight  $u_{ij}$  of each directed edge  $(i, j)$  is a positive integer, called the edge **capacity**.



# The Maximum-Flow Problem

- A digraph satisfying these properties is called a *flow network* or simply a *network*.
- It is assumed that the source and the sink are the only source and destination and all the other vertices are the points where a flow can be redirected without consuming or adding any amount of the material.
- In other words, the total amount of the material entering an intermediate vertex must be equal to the total amount of the material leaving the vertex. This condition is called the *flow-conservation requirement*.
- the flow-conservation requirement can be expressed by the following equality constraint:



# The Maximum-Flow Problem

```
for every edge from  $j$  to  $i$  do //backward edges
  if  $j$  is unlabeled
    if  $x_{ji} > 0$ 
       $l_j \leftarrow \min\{l_i, x_{ji}\}$ ; label  $j$  with  $l_j, i^-$ 
      Enqueue( $Q, j$ )
  if the sink has been labeled
    //augment along the augmenting path found
     $j \leftarrow n$  //start at the sink and move backwards using second labels
    while  $j \neq 1$  //the source hasn't been reached
      if the second label of vertex  $j$  is  $i^+$ 
         $x_{ij} \leftarrow x_{ij} + l_n$ 
      else //the second label of vertex  $j$  is  $i^-$ 
         $x_{ji} \leftarrow x_{ji} - l_n$ 
       $j \leftarrow i$ ;  $i \leftarrow$  the vertex indicated by  $i$ 's second label
    erase all vertex labels except the ones of the source
    reinitialize  $Q$  with the source
return  $x$  //the current flow is maximum
```