HARMONIC FUNCTIONS :

Defn: An expression of the form $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$ is called the Laplace equation in two dimension.

Defn: Any function having continuous second order partial derivatives which satisfies the Laplace equation is called harmonic function.

Defn: Any two harmonic functions u and v such that $f(z)=u+iv$ is analytic are called conjugate harmonic functions.

P.T. the function $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic.

Soln: Let $u=x^3-3xy^2+3x^2-3y^2+1$

$u_x=3x^2-3y^2+6x$; $u_y=-6xy-6y$

$u_{xx}=6x+6$; $u_{yy}=-6x-6=-(6x+6)$

$u_{xx}+u_{yy}=0$

$\therefore$ u satisfies Laplace Eqn.

$\Rightarrow u$ is harmonic.

S.T. the function $u = y + e^x \cos y$ is harmonic

Soln: $u = y + e^x \cos y$

$u_x = e^x \cos y \quad ; \quad u_y = 1 - e^x \sin y$

$u_{xx} = e^x \cos y \quad ; \quad u_{yy} = -e^x \cos y$

$\therefore u_{xx} + u_{yy} = 0$

$\therefore$ u satisfies Laplace eqn $\Rightarrow$ u is harmonic