



SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF INFORMATION TECHNOLOGY

19ITB201 – DESIGN AND ANALYSIS OF ALGORITHMS

II YEAR IV SEM

UNIT-V-COPING WITH THE LIMITATIONS OF ALGORITHM

TOPIC: Approximation Algorithms for NP Hard Problems

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Approximation Algorithms for NP Hard Problems





Approximation Approach



- Difficult optimization problems (i.e., there are no known polynomial-time algorithms for these problems) can be solved approximately by a fast algorithm (i.e., polynomial-time algorithm). The found solution is approximate one, but it is close to the exact solution s^* .
- We would like to know how accurate the approximate solution s_a is. The accuracy of an approximate solution s_a is measured by *accuracy ratio* $r(s_a)$.



Approximation Approach



- For minimization problem (i.e., minimize an objective function f):

$$r(s_a) = \frac{f(s_a)}{f(s^*)},$$

where s^* is an exact solution to the problem.



Approximation Approach



- For maximization problem (i.e., maximize an objective function f):

$$r(s_a) = \frac{f(s^*)}{f(s_a)}$$

where s^* is an exact solution to the problem.

- **Performance ratio** of the algorithm A , denoted R_A , is the lowest upper bound of $r(s_a)$ on all instances.



Nearest Neighbor Algorithm for TSP



Step 1 Choose an arbitrary city as the start.

Step 2 Repeat the following operation until all the cities have been visited: go to the unvisited city nearest the one visited last.

Step 3 Return to the starting city.



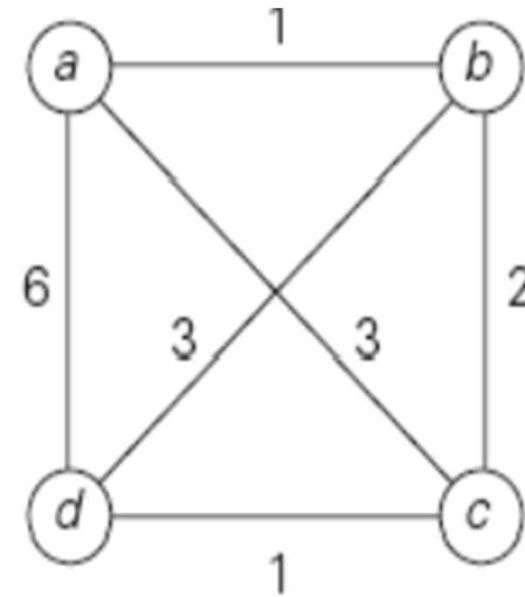
Nearest Neighbor Algorithm for TSP



s_a : $a - b - c - d - a$ of length 10

s^* : $a - b - d - c - a$ of length 8

$$r(s_a) = \frac{f(s_a)}{f(s^*)} = \frac{10}{8} = 1.25$$



(i.e., tour s_a is 25% longer than the optimal tour s^*)



Multifragment-Heuristic Algorithm

Step 1: Sort edges in increasing order of weights.

Initialize the set of tour edges to be constructed to empty set

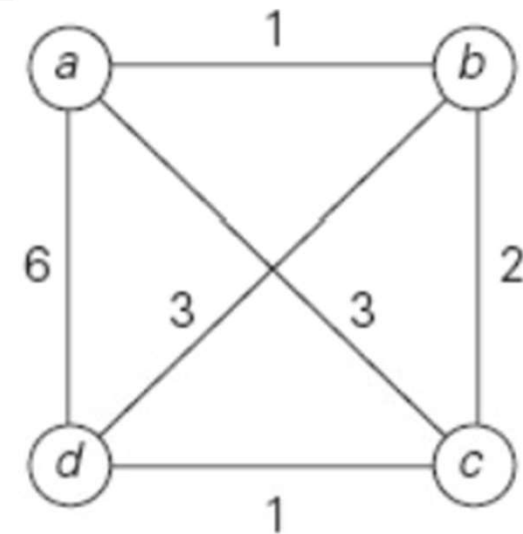
Step 2: Add next edge on the sorted list to the tour, skipping those whose addition would've created a vertex of degree 3 or a cycle of length less than n . Repeat this step until a tour of length n is obtained.

Step 3: Return the set of tour edges



Multifragment-Heuristic Algorithm

$s_a: \{(a, b), (c, d), (b, c), (a, d)\}$





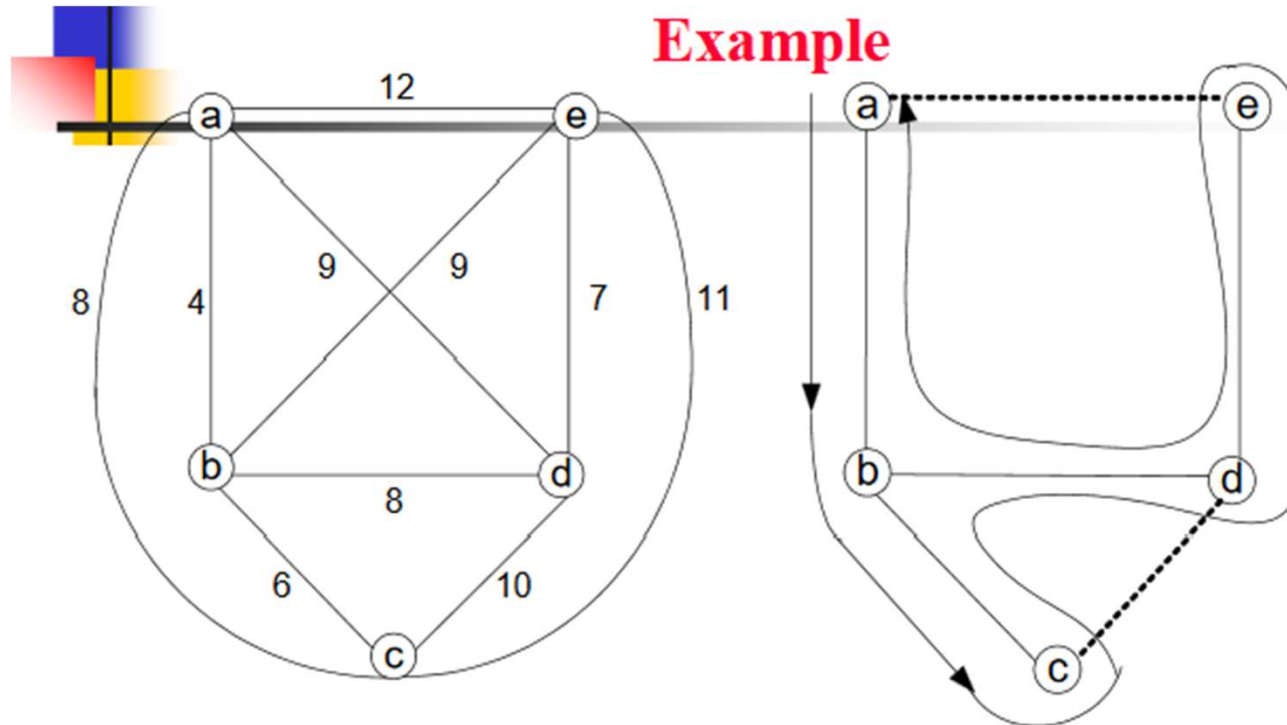
Twice Around the Tree Algorithm



- Step 1:** Construct a minimum spanning tree of the graph (e.g., by Prim's or Kruskal's algorithm)
- Step 2:** Starting at an arbitrary vertex, perform a walk around the minimum spanning tree recording all the vertices passed by.
- Step 3:** Eliminate from the recorded list all repeated occurrences of the same vertex except the starting one at the end of the list. The vertices remaining on the list will form a Hamiltonian circuit.



Twice Around the Tree Algorithm



Walk: $a - b - c - b - d - e - d - b - a$

Tour: $a - b - c - d - e - a$ of length 39

- The tour obtained is not optimal
(better solution: $4 + 6 + 11 + 7 + 9 = 37$).



Assessment



1. In how many directions do queens attack each other?

- a) 1
- b) 2
- c) 3
- d) 4

2. Where is the n-queens problem implemented?

- a) Carom
- b) Chess
- c) Ludo
- d) Cards

Thank You