UNIT - 3

Introduction

If x & y are real numbers then $z = x+iy$ is called a complex number where x is called real part of z, y is called imaginary part of z and the value of i is $\sqrt{-1}$. The complex number $x-iy$ is called the complex conjugate of z and it is denoted by $\bar{z}$. ie) $\bar{z} = x-iy$

Note:

1. $|z| = \sqrt{x^2+y^2}$
2. $|z^2| = z\bar{z}$
3. $z\bar{z} = x^2+y^2 = r^2$
4. $|\bar{z}| = |z|$
5. Real part of $z = \frac{z+\bar{z}}{2}$
6. Imaginary part of $z = \frac{z-\bar{z}}{2}$
7. $z = re^{i\theta}$ is called polar form of z

Function of complex variable

$w = f(z) = u(x,y) + iv(x,y)$ where $u(x,y)$ & $v(x,y)$ are real variables.

Analytic function

A function is said to be analytic at a point if its derivative exists not only that point but also some neighbourhood of that point

Cauchy - Riemann equations (Cartesian Coordinates)

If the function $f(z) = u(x,y) + iv(x,y)$ is analytic in a region R of the z plane, then

i) $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ exists

ii) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ at any point in region