UNIT - III

GEARS AND GEAR TRAINS

Gears are used to transmit motion from one shaft to another or between a shaft and a slide. This is accomplished by successively engaging teeth.

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Gears use no intermediate link or connector and transmit the motion by direct contact. In this method, the surfaces of two bodies make a tangential contact.

Classification of gears

Gears can be classified according to the relative position of their shaft axes as follows.

I) Parallel shafts:

i) Spur gears

ii) Spur rack and pinion

iii) Helical gears

iv) Double-helical and herringbone gears.

II) Intersecting shafts:

i) Spiral bevel gears

ii) Zerol bevel gears.

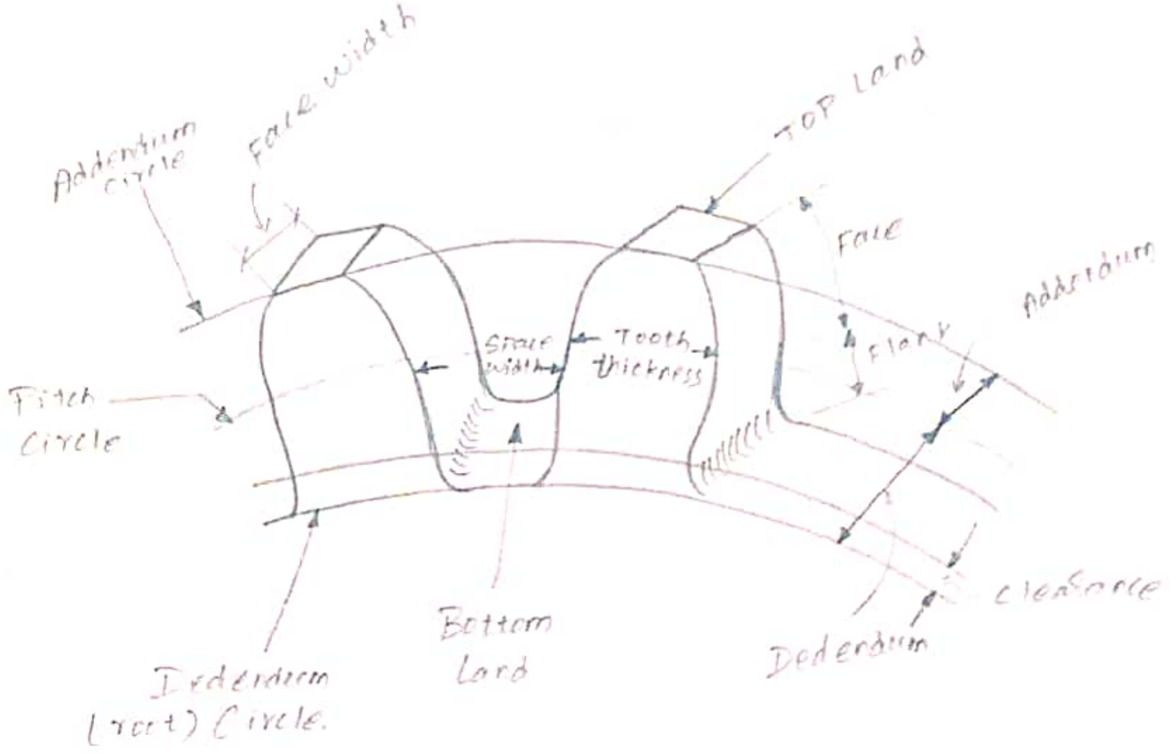
III) Skew shafts:

i) Crossed helical gears

ii) Worm gears

iii) Hypoid gears.

Gear terminology S.C.R.337



Law of toothed gearing

The law of gearing states the Condition which must be fulfilled by gear tooth profiles to maintain a Constant angular velocity ratio between two gears. The law of gearing states that for obtaining Constant velocity ratio, at any instant of teeth, the Common normal at each point should always pass through a pitch point, situated on the line of joining the centre of rotation of the pair of mating gears.

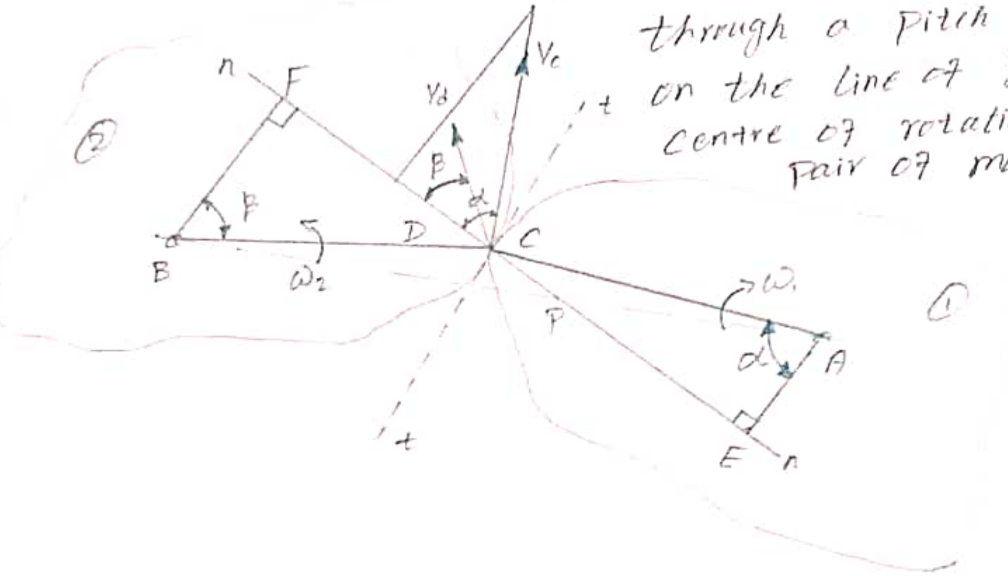


Figure shows two bodies 1 and 2 representing a portion of the two gears in mesh.

A point 'C' on the tooth profile of the gear 1 is in contact with a point 'D' on the profile of the gear 2. The two curves in contact at points C and D must have a common normal at the point.

Let it be n–n.

Let. ω_1 = instantaneous angular velocity of the gear 1. (c.w)

ω_2 = instantaneous angular velocity of the gear 2 (c.c.w)

V_c = Linear velocity of C

V_d = " " " D

Then.
$V_c = \omega_1 \times AC$ in a direction ⊥r to AC.

$V_c = \omega_2 \cdot BD$ " " " BD

If the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent t-t. The relative motion between the surfaces along the common normal n-n must be zero to avoid the separation, of the two teeth into each other.

Component of V_c along n-n $= V_c \cos\alpha$

" " V_d " " $= V_d \cos\beta$

Relative motion along n-n $= V_c \cos\alpha - V_d \cos\beta$

Draw ⊥r. AE and BF on n-n from points A and B. respectively. Then $\angle CAE = \alpha$ and $\angle DBF = \beta$.

For proper contact,

$$V_c \cos\alpha - V_d \cos\beta = 0.$$

$$\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$$

$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \cancel{\cos\beta} \frac{BF}{BD} = 0.$$

$$\omega_1 AE - \omega_2 BF = 0$$

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP}$$ ($\because$ Δ AEP and BFP are similar)

For Constant angular velocity ratio of the two gears, the Common normal at the point of Contact of the two mating teeth must pass through the pitch point.

Also, as the Δs AEP and BFP are similar,

$$\frac{BP}{AP} = \frac{FP}{EP}$$

$$\frac{\omega_1}{\omega_2} = \frac{FP}{EP} \quad (or)$$

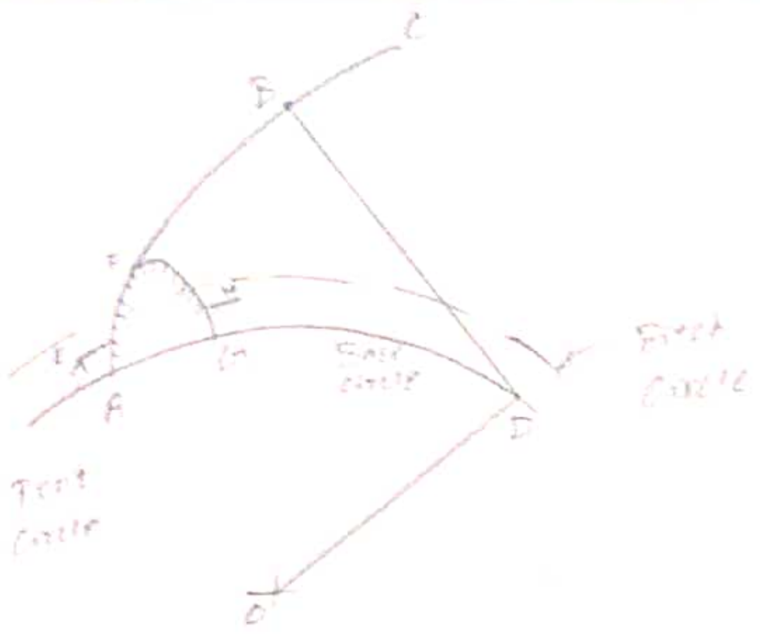
$$\boxed{\omega_1 EP = \omega_2 FP}.$$

Involute gearing: S.S.R. 345

An involute is defined as the locus of a point on a straight line which rolls without slipping on the Circumference of a circle.

Also, it is the path traced out by the end of a piece of taut cord being unwound from the Circumference of a circle.

The Circle on which the Straight line rolls or from which the Cord is unwound is known as the base Circle.



Interchangeable gears

The gears are interchangeable if they are standard ones. It is always a matter of Convenience to have gears of standard dimensions which Can be replaced Easily when they are worn out. The gears are interchangeable if they have.

- The Same module.
- The Same Pressure angle.
- The Same addendums and dedendums
- The Same thickness.

Standard System

A tooth System which relates the various Parameters of gears Such as mentioned above to attain interchangeability of the gears of all teeth numbers but of the Same Pressure angle and pitch is Said to be Standard System.

Non Standard gears SSR 348

The term non standard gears apply to such gears as are modified by changing some standard parameters like pressure angle, addendum, centre distance. These changes are made to improve the performance of the gear operation or from the economical point of view.

For a typical type of teeth, it is observed that if the number of teeth is reduced from a certain number, the problems of interference, undercutting and contact ratio hamper the smooth running of the gears.

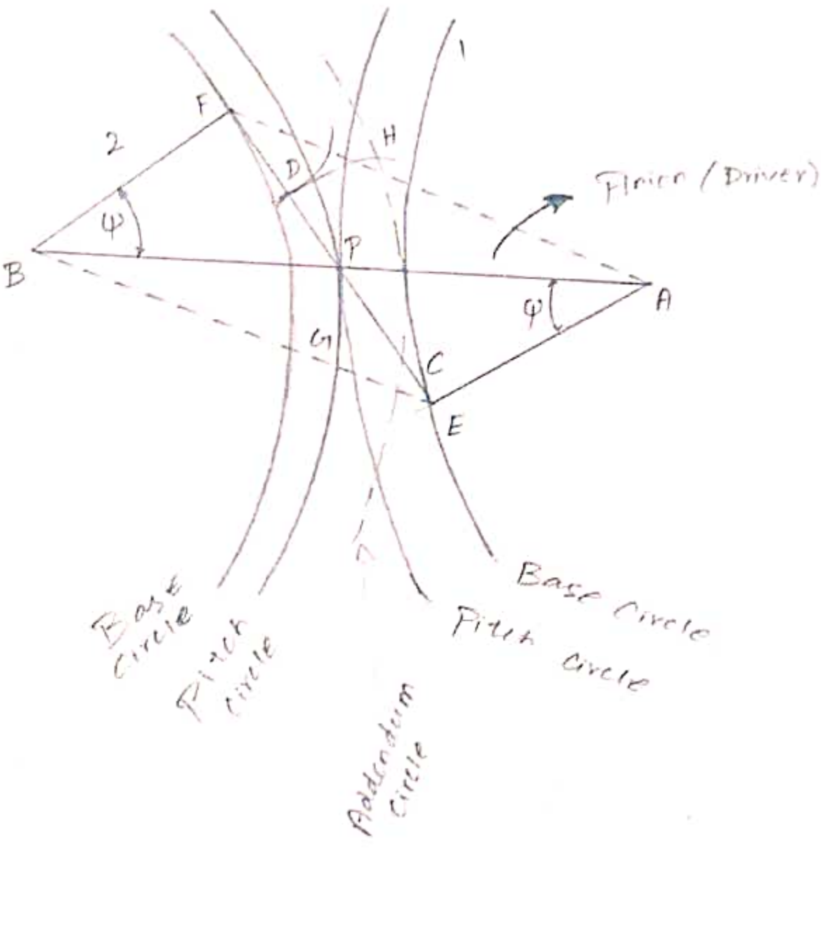
Therefore, the main reason to employ non-standard gears is to prevent interference and undercutting and to maintain a reasonable contact ratio.

Interference in involute gears 354

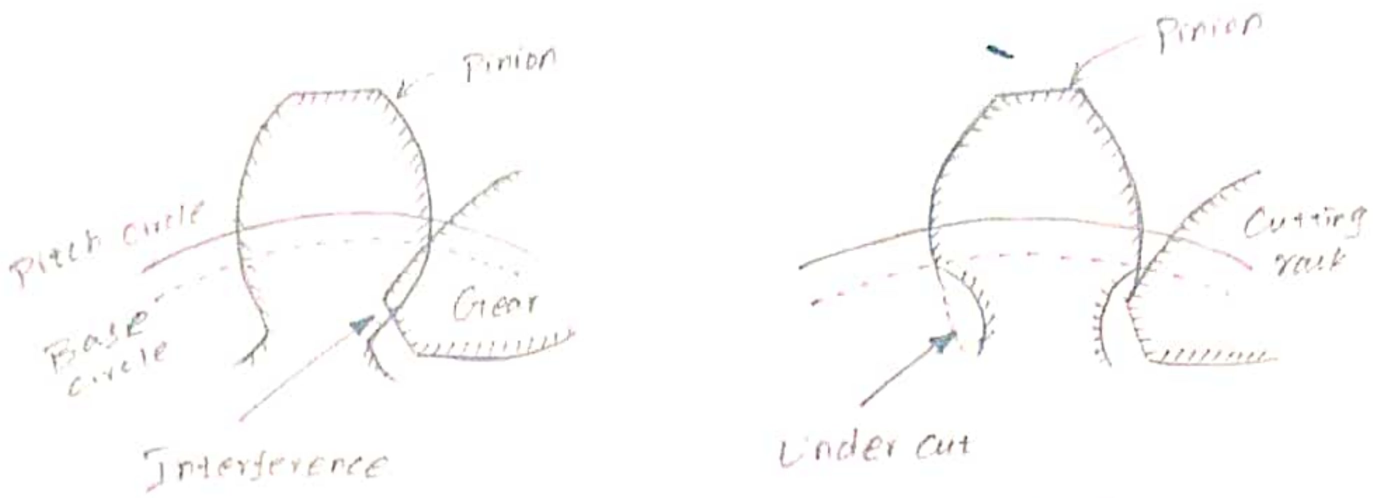
Power transmission through a pair of teeth is along the line of action or the common normal to the two involutes at the point of contact. The common normal is also a common tangent to the two base circles and passes through the pitch point. At any instance, the portions of the tooth profiles which are in contact must be involutes, so that the line of action does not deviate.

If any of the two surfaces is not an involute, the two surfaces would not touch each other tangentially and the transmission of power would not be proper.

Mating of two non-conjugate teeth is known as interference. Because the two teeth do not slide properly and thus rough action and binding occurs.



Undercutting



(a) (b)

Figure shows a Pinion. A portion of ~~the~~ its dedendum falls inside the base Circle. The profile of the tooth inside the bas circle is radial. If the addendum of the mating gear is more than the limiting value, it interferes with the dedendum of the Pinion and the two gears are locked.

A gear having its material removed in this manner is said to be undercut and the process is known as undercutting.

Undercutting will not take place if the teeth are designed to avoid interference.

GEAR TRAINS SSR 378

A gear train is a Combination of gears used to transmit motion from one shaft to another. It becomes necessary when it is required to obtain Large Speed reduction within a Small Space.

Types of gear trains

i) Simple gear trains

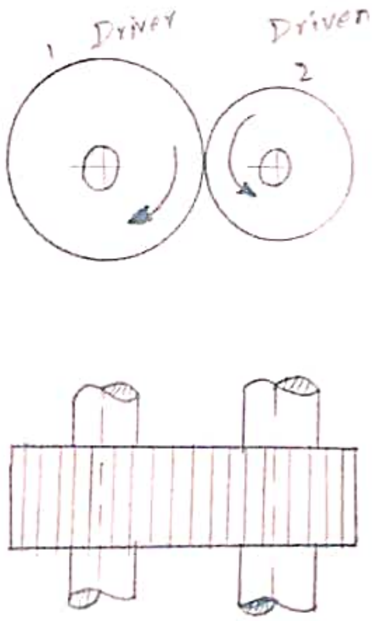
ii) Compound gear trains

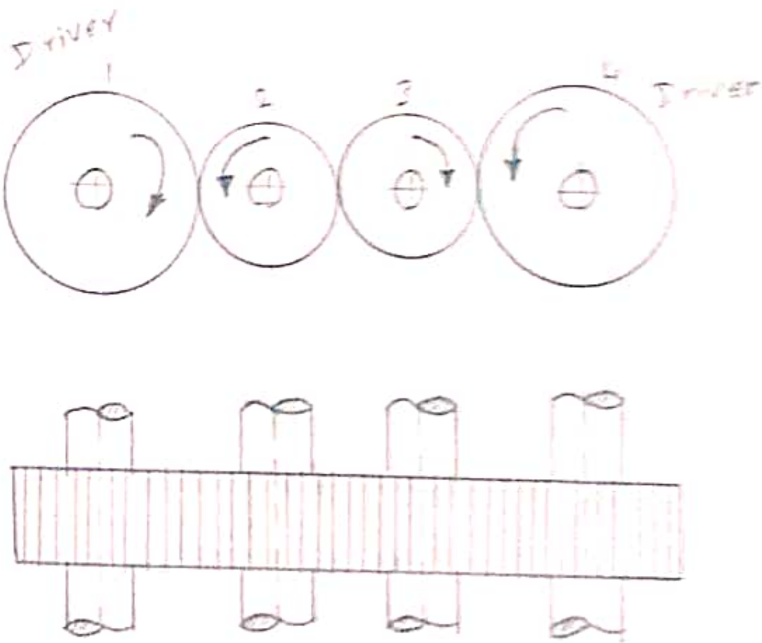
iii) Reverted gear trains

iv) Planetary or Epicyclic gear train

Simple gear trains $\boxed{RSK - 428}$

When there is ~~ety~~ only one gear on Each shaft is exist. it is known as Simple gear train





Let,

N_1 = Speed of gear 1 (or driver) in rpm

N_2 = Speed of gear 2 (or driven) in rpm.

T_1 = No. of teeth in gear 1.

T_2 = " " " 2.

Since, the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their no. of teeth.

$$\therefore \text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}.$$

The ratio of the speed of the driven or follower to the speed of the driver is known as <u>TRAIN VALUE</u> of the gear.

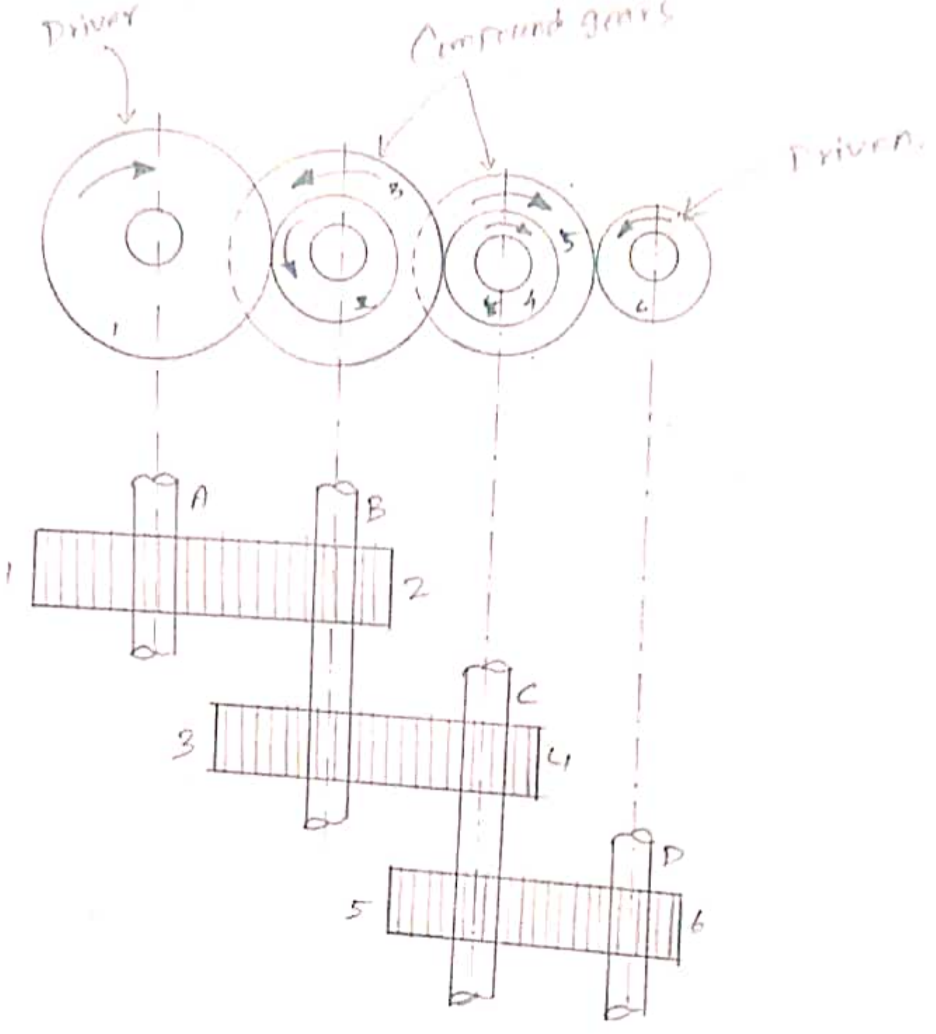
$$\text{Train Value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}.$$

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train Value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven.}}$$

Compound gear train [430]

When there are more than one gear on a shaft in a transmission it is said to be Compound gear train.



Let,

N_1 = Speed of driving gear 1.

T_1 = No. of teeth on driving gear 1.

$N_2, N_3 \ldots N_6$: Speed of respective gears in rpm.

$T_2, T_3 \ldots T_6$: No. of teeth on respective gears

$$\frac{\text{Driver}}{\text{Driven}} \quad \frac{N_1}{N_2} = \frac{T_2}{T_1} \longrightarrow (1)$$

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \longrightarrow (2)$$

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \longrightarrow (3)$$

The speed ratio of Compound gear train is obtained by multiplying the eqns. (1), (2) and (3).

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

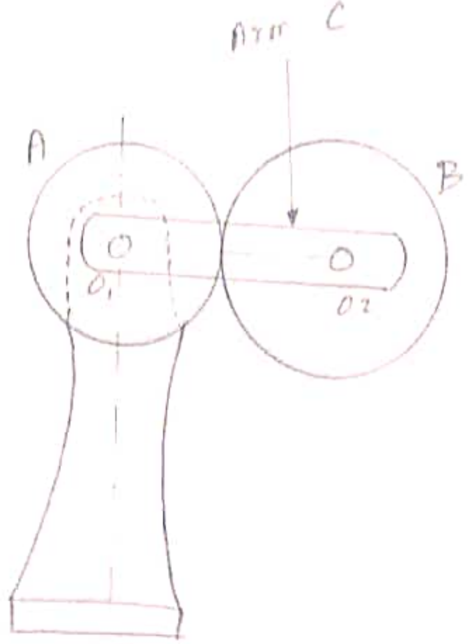
$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven (or) follower}}$$

$$= \frac{\text{Product of No. of teeth on } \boxed{\text{drivens}}}{\text{Product of No. of teeth on } \boxed{\text{drivers}}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the No. of teeth on the } \boxed{\text{drivers}}}{\text{Product of the No. of teeth on the } \boxed{\text{drivens}}}$$

Epicyclic gear train [unit]



In the above figure. gear A and the arm C have a common axis at O_1 about which they can rotate the gear B meshes with gear A and has its axis on the arm at O_2 about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vise-versa.

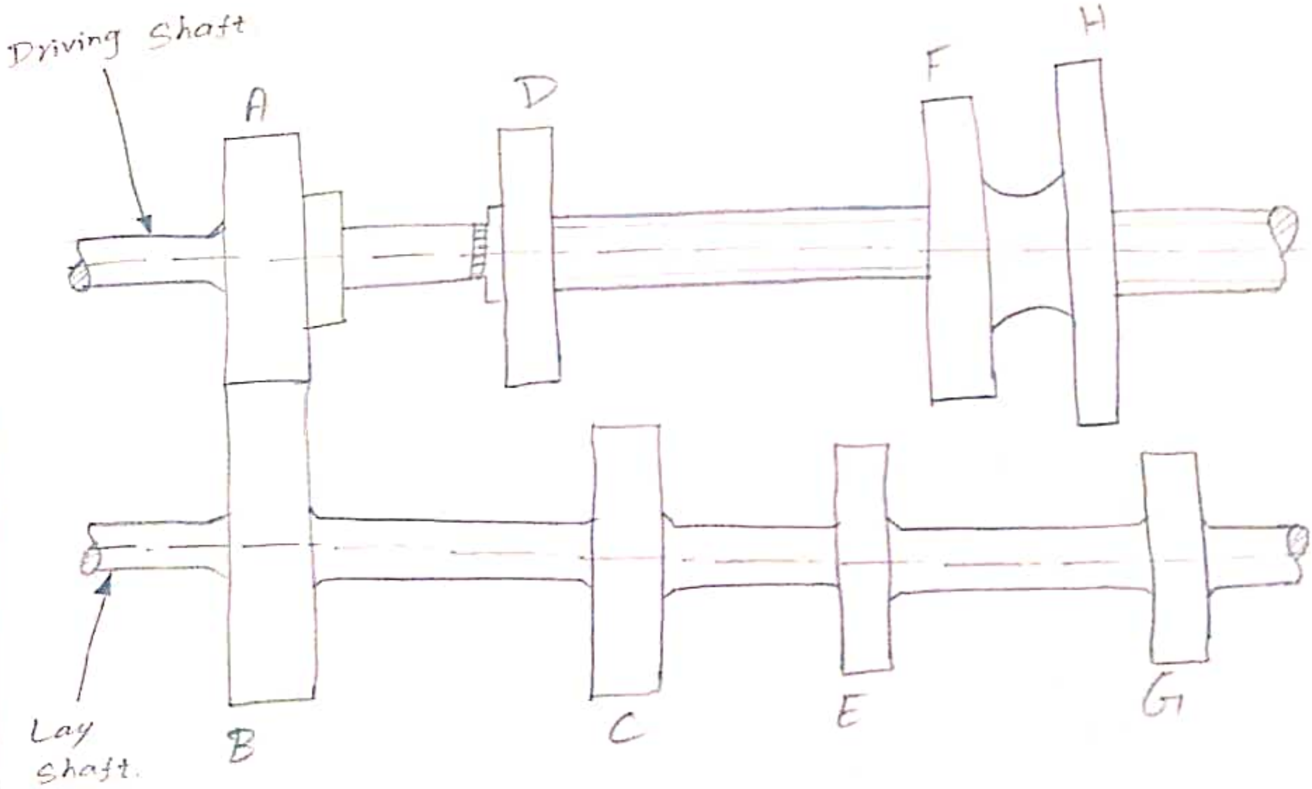
The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.

It is used in the back gear of lathe. differential gears of the automobiles. hoists, pulley blocks, wrist watches.

Automatic transmission gear trains

Gear trains may be used to obtain different speeds of an automobile. A simple sliding gear box makes use of a compound gear train and is engaged by sliding gears on the driven shaft to mesh with the gears on the lay shaft.

Sliding gear box



The above figure shows the gear box is in the neutral position. The pinion 'A' is keyed to the driving shaft is in constant mesh with the gear 'B' on the lay shaft. The gears B, C, E and G are rigidly fixed on the lay shaft.

First gear

The first gear is engaged by the sliding gear 'H' towards right and meshing it with the gear 'G' of the lay shaft.

$$\therefore \text{Train value} = \frac{T_A}{T_B} \times \frac{T_G}{T_H}$$

Second gear

By sliding the gear 'F' towards left and engaging it with the gear 'E' of the lay shaft.

$$\text{Train value} = \frac{T_A}{T_B} \times \frac{T_E}{T_F}$$

Third gear

By sliding the gear 'D' towards the right and engaging it with the gear 'C'.

$$\text{Train value} = \frac{T_A}{T_B} \times \frac{T_C}{T_D}$$

Top gear

The gear 'D' is directly engaged with the gear 'A' through a dog clutch.

Reverse gear

To put the vehicle in the reverse gear an idler is made to mesh with G and H. So that both of them rotate in the same direction. therefore rotating the driven shaft in opposite direction.

A Pinion having 40 teeth drives a gear having 100 teeth. The profile of the gears in involute with 20° pressure angle, 14 mm module and addendum of 12 mm. Find
(i) Length of Path of Contact
(ii) Arc of Contact
(iii) Contact ratio.

SOLUTION. (1) Length of path of Contact.

Pitch circle radius of gear wheel, $\boxed{R = \frac{m\,T_G}{2}}$

$$\therefore R = \frac{14 \times 100}{2} = 700 \text{ mm}$$

Pitch circle radius of pinion, $\boxed{r = \frac{m\,T_P}{2}}$

$$\therefore r = \frac{14 \times 40}{2} = 280 \text{ mm}$$

Addendum radius of gear wheel, $\boxed{R_A = R + \text{addendum}}$

$$R_A = 700 + 12 = 712 \text{ mm}$$

Addendum radius of pinion, $\boxed{r_A = r + \text{addendum}}$

$$r_A = 280 + 12 = 292 \text{ mm}$$

Length of path of approach (KP).

$$\boxed{KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi}$$

$$= \sqrt{(712)^2 - (700)^2 \cdot (\cos 20)^2} - 700 \sin 20$$

$$= \sqrt{74263.11} - 239.41$$

$$= 272.5 - 239.4$$

$$\boxed{KP = 33.11 \text{ mm}}$$

Length of path of recess, $\boxed{(PL) = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi}$

$$PL = \sqrt{292^2 - (280)^2 \cdot (\cos 20)^2} - 280 \sin 20$$

$$= \sqrt{16035} - 95.7$$

$$= 126.63 - 95.7$$

$$\boxed{PL = 30.93 \text{ mm}}$$

∴ Length of path of contact $(KL) = KP + PL$

$$\therefore KL = 33.11 + 30.93$$

$$\boxed{KL = 64.04 \text{ mm}}$$

ii) Length of arc of contact

$$\boxed{L.o.a.c = \frac{\text{Length of path of contact}}{\cos\phi}}$$

$$= \frac{64.04}{\cos 20}$$

$$\boxed{L.o.a.c = 68.15 \text{ mm}}$$

iii) Contact ratio:

$$\boxed{C.r = \frac{\text{Length of arc of contact}}{\text{circular pitch } (P_c)}}$$

$P_c = \pi . m$

$$= \frac{68.15}{\pi \times m} = \frac{68.15}{\pi \times 14} = 1.55$$

$$\boxed{C.r = 1.55 \simeq 2}$$

RESULT:

i) Length of path of contact = 64.04 mm

ii) Length of arc of contact = 68.15 mm

iii) Contact ratio = 2

The following data refers to two matching involute gears of 20° pressure angle

i) Number of teeth on pinion = 20

ii) Gear ratio = 2

iii) Speed of pinion = 250 rpm

iv) Module = 12 mm

If the addendum of each wheel is such that the path of approach and path of recess on each side are half the maximum possible length. Find

(i) Addendum of both the wheels

(ii) The length of arc of contact

(iii) Contact ratio

(iv) Maximum sliding velocity

(v) Angle turned through by the pinion

(vi) Angle turned through by the gear wheel when one pair of teeth is in contact.

SOLUTION:

$T_P = 20$, $N_P = 250$ rpm. $m = 12$ mm, $G = 2 = \frac{T_G}{T_P}$

Angular velocity of pinion,

$$\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 250}{60} = 26.16 \text{ rad/s.}$$

Gear ratio, $G = \frac{T_G}{T_P} = 2$.

$$\therefore T_G = 2 \times T_P = 2 \times 20 = 40.$$

$$\boxed{G = \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P}} = 2$$

$$\therefore \omega_G = \frac{\omega_P}{2} = \frac{26.16}{2} = 13.08 \text{ rad/s.}$$

Pitch Circle radii of pinion and gear wheel is,

$$r = \frac{m T_P}{2} = \frac{12 \times 20}{2} = 120 \text{ mm}$$

$$R = \frac{m T_G}{2} = \frac{12 \times 40}{2} = 240 \text{ mm.}$$

1) Addendum for both the wheels

Given as addendum on pinion and gear is such that, path of approach and recess are half of their maximum possible values.

$$\therefore \sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi = \frac{r\sin\phi}{2} \longrightarrow (1)$$

$$\sqrt{r_A^2 - r^2 \cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2} \longrightarrow (2)$$

$$\therefore \sqrt{R_A^2 - 240^2 \cdot \cos^2 20^\circ} - 240\sin 20^\circ = \frac{120 \sin 20^\circ}{2}$$

$$\boxed{R_A = 247.78 \text{ mm}}$$

$\therefore$ Addendum of gear wheel $= R_A - R$

$= 247.78 - 240$

$= 7.78$ mm.

Substitutes the R and r values in eqn (2)

$$\sqrt{r_A^2 - 120^2 \cos^2 20^\circ} - 120 \sin 20^\circ = \frac{240 \sin 20}{2}$$

$$\boxed{r_A = 139.5 \text{ mm}}$$

$\therefore$ Addendum of pinion $= r_A - r$

$= 139.5 - 120$

$= 19.5$ mm

ii) Length of arc of Contact

Length of path of approach, $KP = \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$

$$KP = \sqrt{(247.7)^2 - 240^2 \cdot \cos^2 20} - 240\sin 20$$

$$KP = 20.52 \text{ mm}$$

Length of path of recess, $PL = \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

$$PL = \sqrt{(139.5)^2 - 120^2\cos^2 20} - 120\sin 20$$

$$PL = 41.08 \text{ mm}$$

$$KL = KP + PL$$

$$= 20.52 + 41.08$$

$$KL = 61.6 \text{ mm.}$$

$$\therefore L.a.o.c = \frac{KL}{\cos\phi} = \frac{61.56}{\cos 20} = 65.5 \text{ mm}$$

iii) Contact ratio (or) No. of Pair of teeth in Contact

$$C.R = \frac{L.o.a.c}{\pi m} = \frac{65.5}{\pi \times 12} = 1.74 \simeq 2.$$

iv) Maximum Sliding velocity

$$V_s = (\omega_1 + \omega_2) \times PL \quad \Big| \text{ when } PL > KP$$

$$= (26.16 + 13.08) \times 41.08 = 1611.9 \text{ mm/s}$$

$$\boxed{V_s = 1.61 \text{ m/s}}$$

v) Angle turned through by Pinion

$$= \frac{\text{Length of arc of Contact}}{\underset{(2\pi \times r)}{\text{Circumference of Pinion}}} \times 360^\circ$$

$$= \frac{65.51 \times 360^\circ}{2\pi \times 120}$$

2, 3, 7, 12, 26, 15.
31, 56.
105, 109.

$$= \underline{31.28^\circ}$$

vi) Angle turned through by the gear wheel

$$= \frac{\text{Length of arc of Contact}}{\underset{(2\pi \times R)}{\text{Circumference of gear wheel}}} \times 360^\circ$$

$$= \frac{65.5 \times 360^\circ}{2\pi \times 240}$$

$$= \underline{15.64^\circ}$$