Inductive and Capacitive elements

An alternating quantity can be represented using

i) Waveform

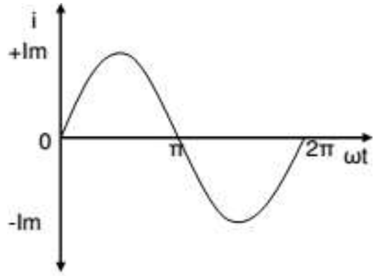
ii) Equations

iii) Phasor

A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.

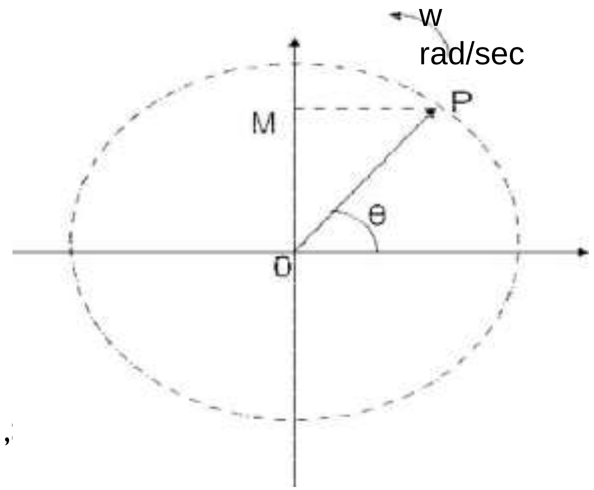
Inductive and Capacitive elements



$$i = I_m \sin \omega t$$

In phasor form the above wave is written as $\bar{\ } = I_m \angle 0^0$

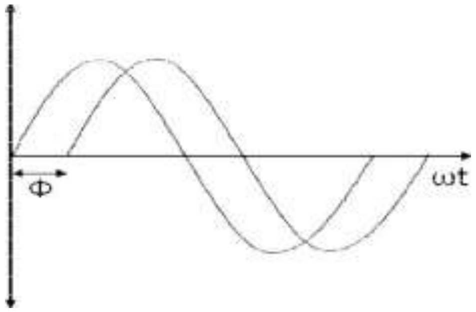
Draw a line OP of length equal to I_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by OM=OPsinθ = $I_m$sinωt. Hence the line OP is the phasor representation of the sinusoidal current.



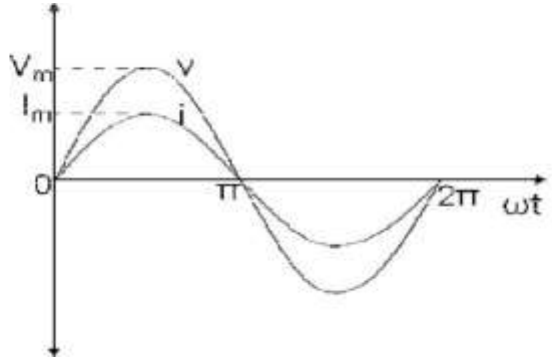
Inductive and Capacitive elements

In phase –

Two waveforms are said to be **in phase**, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.



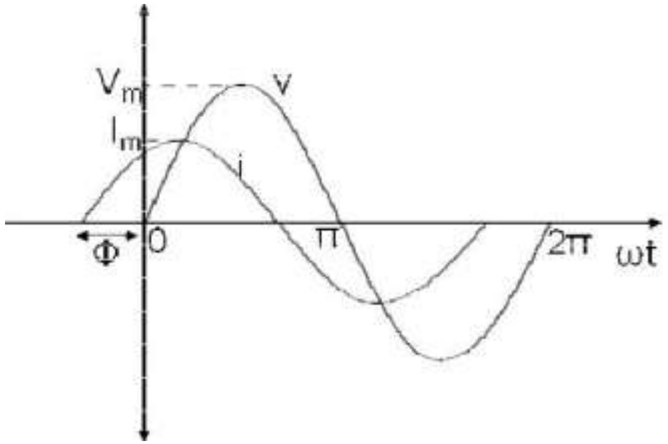
$$v = v_m \sin(\omega t)$$
$$i = i_m \sin(\omega t)$$

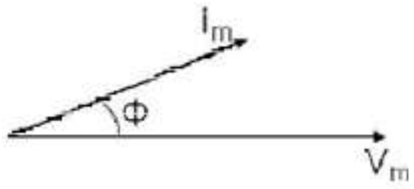


Inductive and Capacitive elements

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown



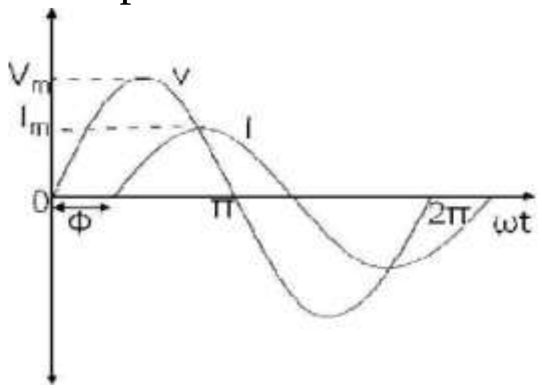


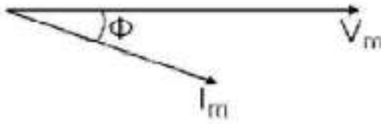
$$v = v_m \sin(\omega t) => \bar{} = V_m \angle 0^0$$
$$\mathrm{i} = \mathrm{i}_m \sin(\omega t + \theta) => \bar{} = I_m \angle \theta^0$$

Inductive and Capacitive elements

lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation





$$v = v_m \sin(\omega t) => \bar{} = V_m \angle 0^0$$
$$\mathrm{i} = \mathrm{i}_m \sin(\omega t - \theta) => \bar{} = I_m \angle - \theta$$

AC Circuit with resistor, inductor and capacitor

AC Circuit with resistor

Let an alternating source of emf be connected across a resistor of resistance R.
The instantaneous value of the applied emf is
$e = E_o \sin \omega t$ . . .(1)

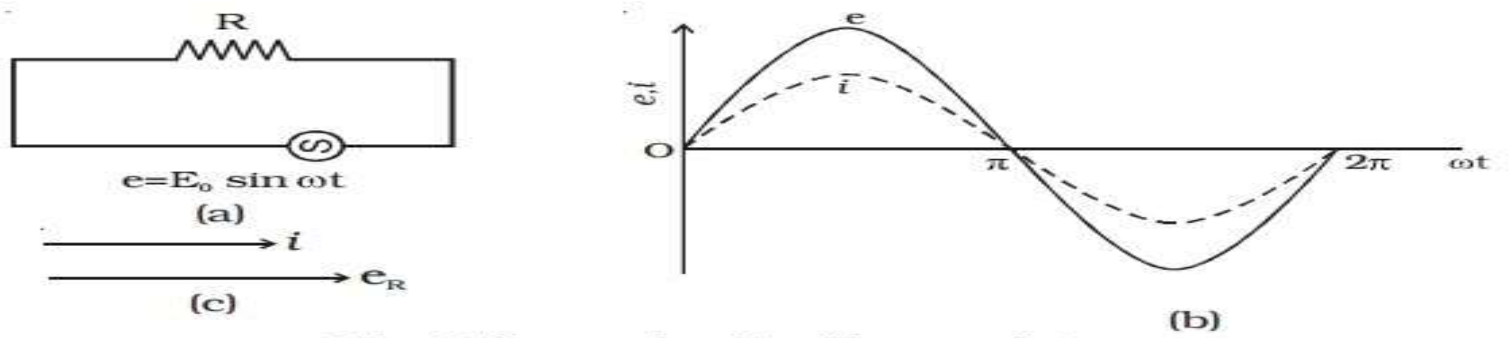


Fig 4.18 a.c. circuit with a resistor

If i is the current through the circuit at the instant t, the potential drop across R is, $e = iR$

Potential drop must be equal to the applied emf.
Hence, $iR = E_o \sin \omega t$

$i = \frac{E_o}{R} \sin \omega t$; $i = I_o \sin \omega t$ ...(2)

where $I_o = \frac{E_o}{R}$, is the peak value of a.c in the circuit. Equation

In a resistive circuit, the applied voltage and current are in phase with each other

Inductive and Capacitive elements

AC Circuit with an inductor

Let an alternating source of emf be applied to a pure inductor of inductance L. The inductor has a negligible resistance and is wound on a laminated iron core. Due to an alternating emf that is applied to the inductive coil, a self induced emf is generated which opposes the applied voltage. (eg) Choke coil.

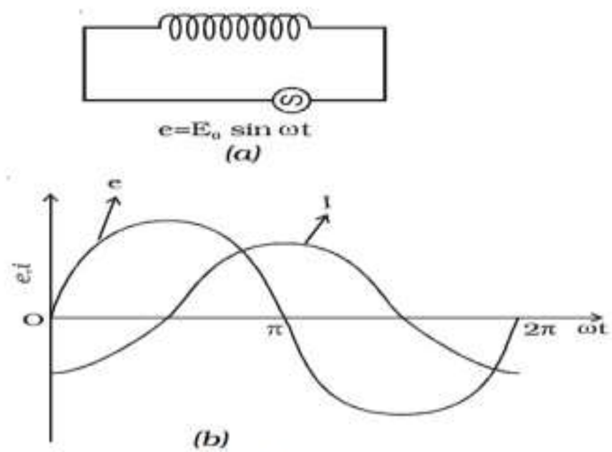


Fig 4.19 Pure inductive circuit

Therefore $e = -e'$

$$E_o \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad E_o \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{E_o}{L} \sin \omega t \, dt$$

Integrating both the sides

$$i = \frac{E_o}{L} \int \sin \omega t \, dt$$

$$= \frac{E_o}{L}\left[-\frac{\cos \omega t}{\omega}\right] = -\frac{E_o \cos \omega t}{\omega L}$$

$$i = \frac{E_o}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = I_o \cdot \sin\left(\omega t - \frac{\pi}{2}\right) \quad \ldots(2)$$

where $I_o = \frac{E_o}{\omega L}$. Here, ωL is the resistance offered by the coil. It is called inductive reactance. Its unit is ohm .

Inductive and Capacitive elements

From equations (1) and (2) it is clear that in an a.c. circuit containing a pure inductor the current i lags behind the voltage e phase angle of $\pi/2$.

Conversely the voltage across L leads the current by the phase angle of $\pi/2$. This fact is presented graphically in Fig 4.19b.

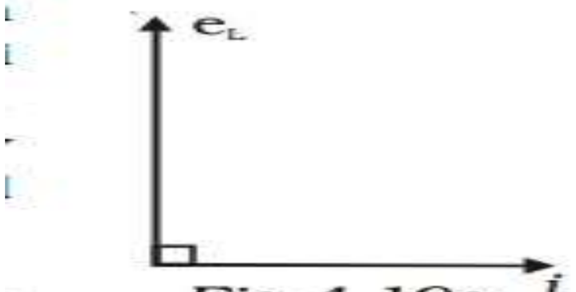


Fig 4.19c
Phasor diagram

Fig 4.19c represents the phasor diagram of a.c. circuit containing only L.

Inductive reactance

$X_L = \omega L = 2\pi \nu L$, where ν is the frequency of the a.c. supply

For d.c. $\nu = 0$; $\therefore X_L = 0$

Thus a pure inductor offers zero resistance to d.c. But in an a.c. circuit the reactance of the coil increases with increase in freque

Pure Capacitive

AC Circuit with a capacitor

An alternating source of emf is connected across a capacitor of capacitance C (Fig 4.20a). It is charged first in one direction and then in the other direction.

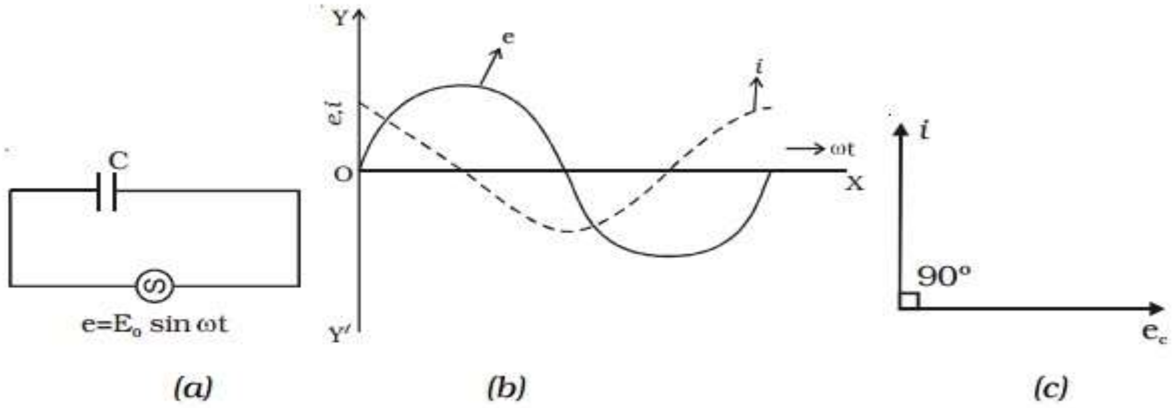


Fig 4.20 Capacitive circuit

Pure Capacitive circuit

The instantaneous value of the applied emf is given by

$$e = E_o \sin \omega t \quad ...(1)$$

At any instant the potential difference across the capacitor will be equal to the applied emf

$\therefore$ $e = q/C$, where q is the charge in the capacitor

But $$i = \frac{dq}{dt} = \frac{d}{dt}(Ce)$$

$$i = \frac{d}{dt}(C\ E_o \sin \omega t) = \omega\ CE_o.\ \cos \omega t$$

$$i = \frac{E_o}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = I_o \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(2)$$

where $I_o = \frac{E_o}{(1/\omega C)}$

$\frac{1}{\omega C} = X_C$ is the resistance offered by the capacitor. It is called capacitive reactance. Its unit is ohm .

Pure Capacitive circuit

From equations (1) and (2), it follows that in an a.c. circuit with a capacitor, the current leads the voltage by a phase angle of π/2. In other words the emf lags behind the current by a phase angle of π/2. This is represented graphically in Fig 4.20b.

Fig 4.20c represents the phasor diagram of a.c. circuit containing only C.

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

where ν is the frequency of the a.c. supply. In a d.c. circuit

$\nu = 0$

$$\therefore \quad X_C = \infty$$

Thus a capacitor offers infinite resistance to d.c. For an a.c. the capacitive reactance varies inversely as the frequency of a.c. and also inversely as the capacitance of the capacitor.