NUMERICAL INTEGRATION BY SIMPSON'S 1/3 RULE

SIMPSON'S 1/3 RULE:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$= \frac{h}{3}\left[A+4B+2C\right]$$

where A = Sum of the first & last ordinates

B = Sum of the odd ordinates

C = Sum of the even ordinates

(ie) an even number of equal subintervals.

Dividing the range into 10 equal parts, find the value of $\int_0^{\pi/2} \sin x\,dx$ by Simpson's 1/3 rule

Soln:

x :	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y=\sin x$:	0	0.1564	0.3090	0.4540	0.5878	0.7071

x :	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$
$y:\sin x$:	0.8090	0.8910	0.9511	0.9877	1

By Simpson's 1/3 rule,

$$\int_0^{\pi/2} \sin x\,dx = \frac{h}{3}\left[(y_0+y_{11})+4(y_1+y_3+y_5+y_7+y_9)+2(y_2+y_4+y_6+y_8+y_{10})\right]$$

$$=\frac{\pi}{20}\cdot\frac{1}{3}\left[(0+1)+4(3.1962)+2(2.6569)\right]$$

$$=1.0000$$

Find the value of $\log_e 5$ from $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $1/3$ rule $(n=10)$.

Soln: Here $y(x) = \frac{1}{4x+5}$

$$h = \frac{5-0}{10} = \frac{1}{2} = 0.5$$

x :	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y :	0.2	0.1429	0.1111	0.0909	0.0769	0.0667	0.0588	0.0526	0.047	0.0434	0.04

By Simpson's $1/3$ rule,

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

$= \frac{1}{6}[2.4148]$

$= 0.4025$ —— (1)

$$\int_0^5 \frac{dx}{4x+5} = \frac{\log(4x+5)}{4}\Big|_0^5$$

$= \frac{1}{4}(\log 25 - \log 5)$

$= \frac{1}{4}\log\left(\frac{25}{5}\right)$

$= \frac{1}{4}\log 5$ —— (2)

From (1) & (2)

$\Rightarrow \frac{1}{4}\log 5 = 0.4025$ $\qquad \log_e$

$\Rightarrow \log 5 = 1.61$

Using Simpson's 1/3 rule, Evaluate $\int_0^{1.2} e^{-x^2}dx$, taking $h = 0.2$.