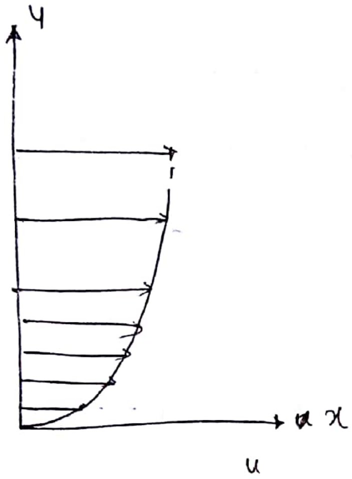
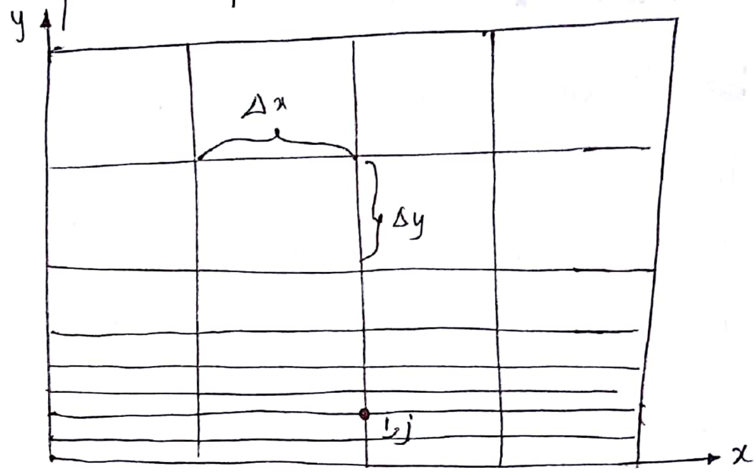
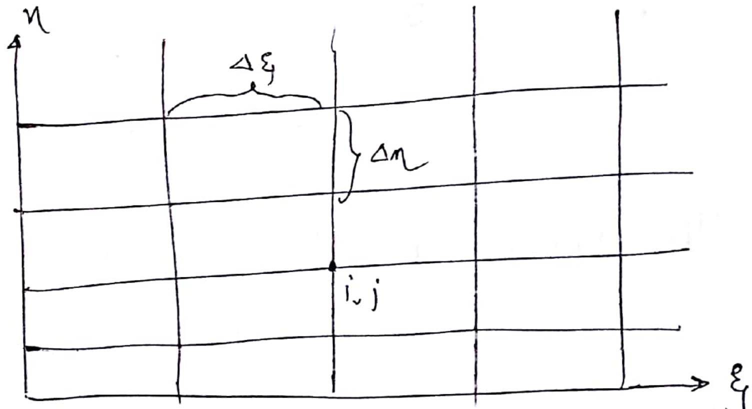
Applications of Grid Streching

(Streched or Compressed Grids)





(a)



(b)

(i) Example of Grid streching
(a) Physical Plane
(b) Computational Plane.

Consider the Physical & computational planes as shown in fig above. Assume we are dealing with viscous flows over a flat surface, where the velocity varies rapidly near the surface as shown in velocity profile sketched at left of physical plane in fig above.

To calculate details of flow near the surface, a finely spaced grid in y-direction should be used as sketched in physical plane. However far away from surface the grid can be coarser. Thus a proper grid should be one in which horizontal lines (co-ordinate) becomes progressively more closely spaced in vertical direction as surface is approached

Wishing to deal with uniform grid in computational plane as shown in fig b

~~We~~ It can be noticed that grid in physical plane is streched as if it is a uniform grid were drawn on a piece of rubber & then the upper portion of rubber were streched in upward direction y

A simple analytical transformation which can accomplish this grid streching is

$$\xi = x \quad \rightarrow (1)$$

$$\eta = \ln(y+1) \quad \rightarrow (2)$$

The inverse transformation is

$$x = \xi \quad \rightarrow (3)$$

$$y = e^{\eta} - 1 \quad \rightarrow (4)$$

Examining above eqs. & the fig. In both physical & computational planes, the vertical grid lines are uniformly spaced in x-direct-ion that is reflected by eqs (1) & (3).

In physical plane Δx is same throughout. In computational plane $\Delta \xi$ is same throughout. $\therefore \Delta x = \Delta \xi$. Thus grid is not streched in x-direction.

But for horizontal lines are uniformly spaced in the computational plane by intent; we stipulate that $\Delta \eta$ is same everywhere. But Δy is given as

$$y = e^{\eta} - 1$$

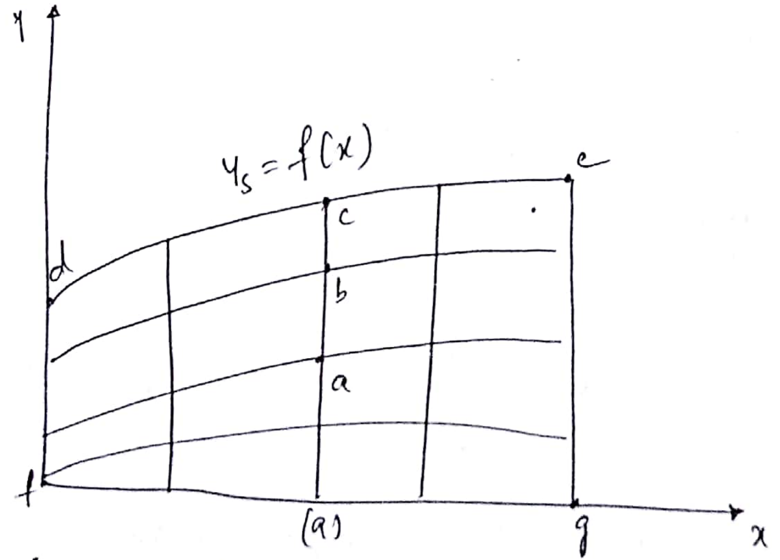
$$\frac{dy}{d\eta} = e^{\eta} d\eta$$

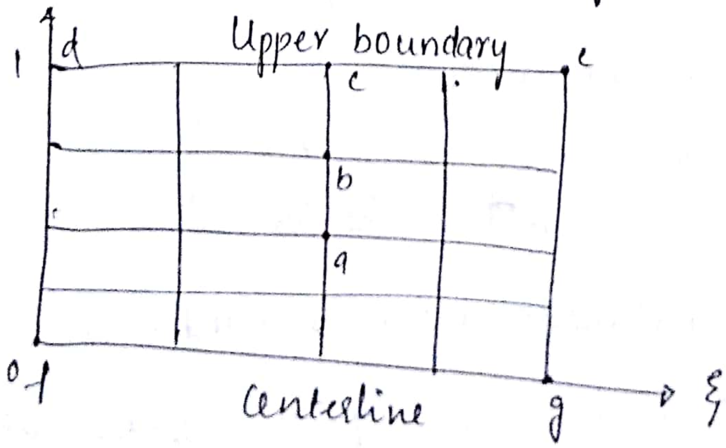
Replacing dy & $d\eta$ with finite increments $\rightarrow \Delta y = e^{\eta} \Delta \eta$

As η becomes larger as we move further above the plate. The value of Δy becomes progressively larger.
Direct or Inverse transformation is mechanism by which streched grids are generated.

Discussion on some of important topics asked in External Exam paper. Can be related to both 5th & 6th unit, could be asked in any of the units (5 or 6th).

Explain ~~body~~ boundary-fitted co-ordinate system for an divergent duct ?





(b)

Consider a flow through divergent duct as shown in fig (a). Curve de is upper wall of the duct, life fg is centerline. A simple rectangular grid in physical plane is not ideal for this case so we have a curvilinear grid which allows both upper boundaries de & the centerline fg to be the co-ordinate lines exactly filling the boundaries.

The curvilinear grid has to be transformed into a rectangular grid in computational plane. To achieve this we have

$$y_s = f(x) \rightarrow (1)$$ be the ordinate of upper surface de.

Following transformation will result in rectangular grid in (ξ, η) space

$$\xi = x \rightarrow (2)$$

$$\eta = \frac{y}{y_s} \quad \text{where } y_s = f(x) \rightarrow (3)$$

For Ex:- Consider point d in physical plane $y = y_d = y_s(x_d)$ when it is substituted in eq.(3)

we have $$\eta_d = \frac{y}{y_s} = \frac{y_s(x_d)}{y_s(x_d)} = 1 \rightarrow (4)$$

Hence in computational plane $\eta = \eta_d = 1$.

Similarly moving to point C $y = y_c = y_s(x_c)$. The co-ordinate in point C is different from point d ; $y_c > y_d$

When y_c substituted in eq. 3

$$\eta_c = \frac{y}{y_s} = \frac{y_s(x_c)}{y_s(x_c)} = 1 \rightarrow (5)$$