# Elliptic Equations

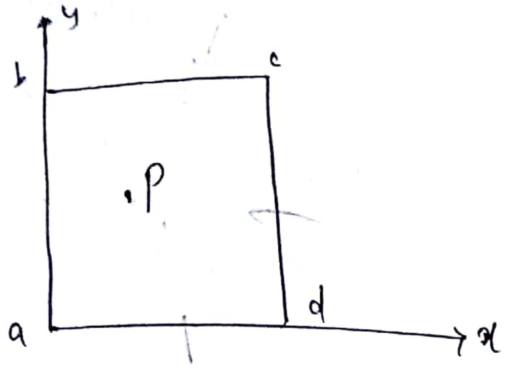


Fig 12 :- Domain & boundaries for solution of elliptic eqs in 2 dimensions.

Consider an elliptic equation in two independent variables x & y. The xy plane is sketched as shown in fig 12.

For elliptic eqs, the characteristic curves are imaginary thus the method of characteristics is useless for elliptic eqs.

For elliptic eqs, there is no limited regions of influence or domain of dependence rather information is propogated everywhere in all directions.

Consider a point in xy plane as in fig 12. Domain of problem is defined by ☐[le] abcd. & P is located inside the domain.

It is in contrast to open domains as seen in hyperbolic & parabolic ~~condition~~ eqs.

If we jab at point P with a needle, i.e. a disturbance is introduced then the disturbance is felt everywhere as per the mathematical characteristic of elliptic eqs.

Point P influences all points in the domain & then in turn the solution at P is influenced by the entire closed boundary abcd. Thus solution at P must be

Carried out simultaneously with the solutions at other points in dome. Thus it in stark contrast to the "marching" solutions considered or obtained to parabolic or hyperbolic ~~eqn~~ eqs.

Thus, Problems involving elliptic eqs are called jury problems, because the solution within the domain depends on the total boundary domain, boundary conditions must be applied over the entire boundary. Boundary

The BC's take following form

1) A specification of u & v (dependent variables) along the boundary. ⟶ Dirichlet condition.
2) A specification of derivatives of dependent variables such as $(\partial v/\partial x)$ along the boundary → Neumann Condition.
3) A mix of both Dirichlet & Neumann conditions.

## Steady, Subsonic, Inviscid flow

In subsonic flow disturbances can move upstream for as far as they want, a finite disturbance in an inviscid subsonic flow will propogate to infinity in all directions.

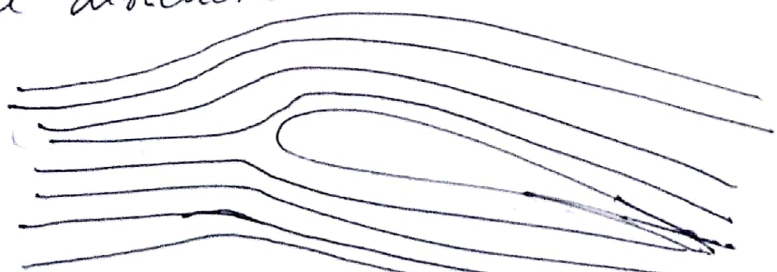

Fig 13:- Smoke flow Photograph of low speed subsonic flow over airfoil.

As seen in fig streamlines in front of airfoil are deflected upwards & those behind airfoil are deflected downwar

The disturbance introduced by presence of airfoil is felt throughout the flow field in case of a subsonic flow as show in fig 13

~~Steady Eulers eq are elliptic when local Mach no is less than unity. Hence an inviscid flow is governed by Eulers eqs. The pic shown in fig 13 is~~

The fig 13 is a physical picture consistent with mathematical behaviour of elliptical eqs. Inviscid flow is governed by Eulers eq. The steady state Eulers eq are elliptical when local Mach no is < 1.

## Incompressible Inviscid flow:

Incompressible flow is a limiting case of a subsonic flow wherein Mach No goes to zero.

Mach No = $V/a$ → (a is speed of sound)

Theoretically for an incompressible flow, compressibility is zero. hence speed of sound (a) is infinite

If $a = \infty$, (very large) $M = 0$, even V is finite

Thus there is no surprise that incompressible, inviscid flow is governed by elliptical eqs.

Refer problem No 3.1 → pg.No (103-105)