CASE STUDY (1).

Steady, Inviscid, Supersonic flow.

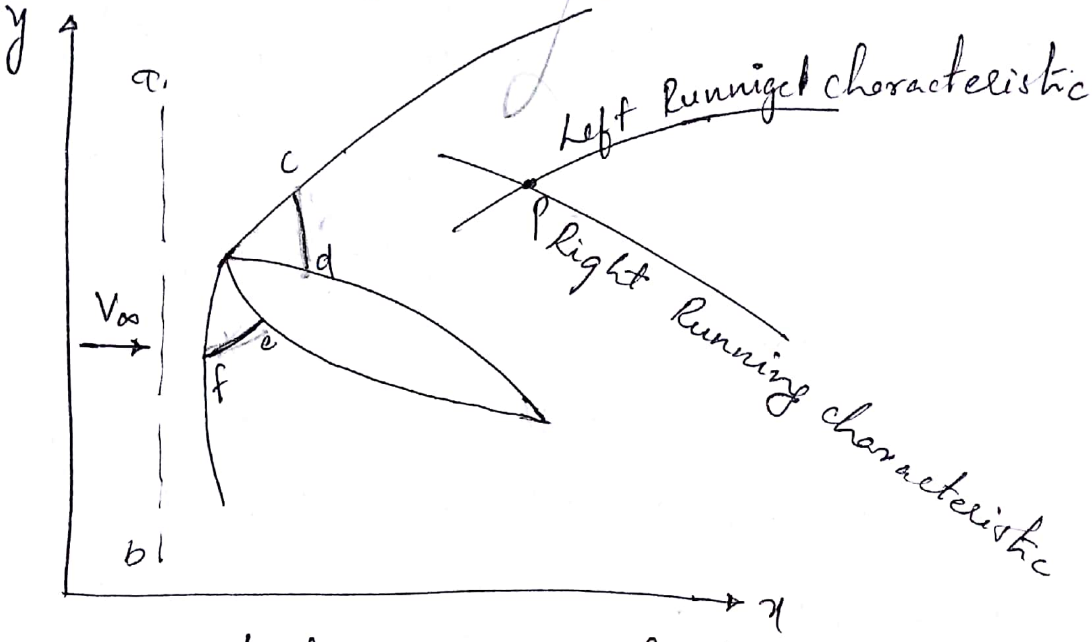

Fig:- 3 Illustration of intial data line for method of characteristics.

Imagine a supersonic flow over a 2D- circular arc-airfoil as shown in fig 3, the airfoil is at α, but α must not be large enough so as to cause leading edge shock wave to become detached, or else there will be pockets of locally subsonic flow.

Consider Euler's eq for steady flow written in conservative or non- conservative form, these eqs are hyperbolic when local Mach No is supersonic. ($M_\infty > 1$)

Hence in figure above when flow is considered to be supersonic everywhere, the entire flow field is governed by hyperbolic eqs.

General flow direction is x. Thus a flow field can be computed with given intial data at some location in the flow & then solving governing eqs, numerically marching step by step.

The location of intial data line is influenced by shock fitting or shock capturing used in calculation.

If shock capturing is used, line ab stream can be used as intial data line, where intial data are simply free stream conditions along ab.

If shock fitting is used lines cd & ef just downstream of the nose & reaching across the flow field from body surface to shock surface can be used as intial data lines.

In this case the intial data specified along cd & ef are associated with classic solution of the oblique shock flow over a wedge.

Wedge Angle is equal to body angle at the nose relative to the freestream direction.

It yields classical solution with constant properties along CD & another set of constant properties along ef.

The remainder of flow field is calculated by marching downstream of these intial data lines.

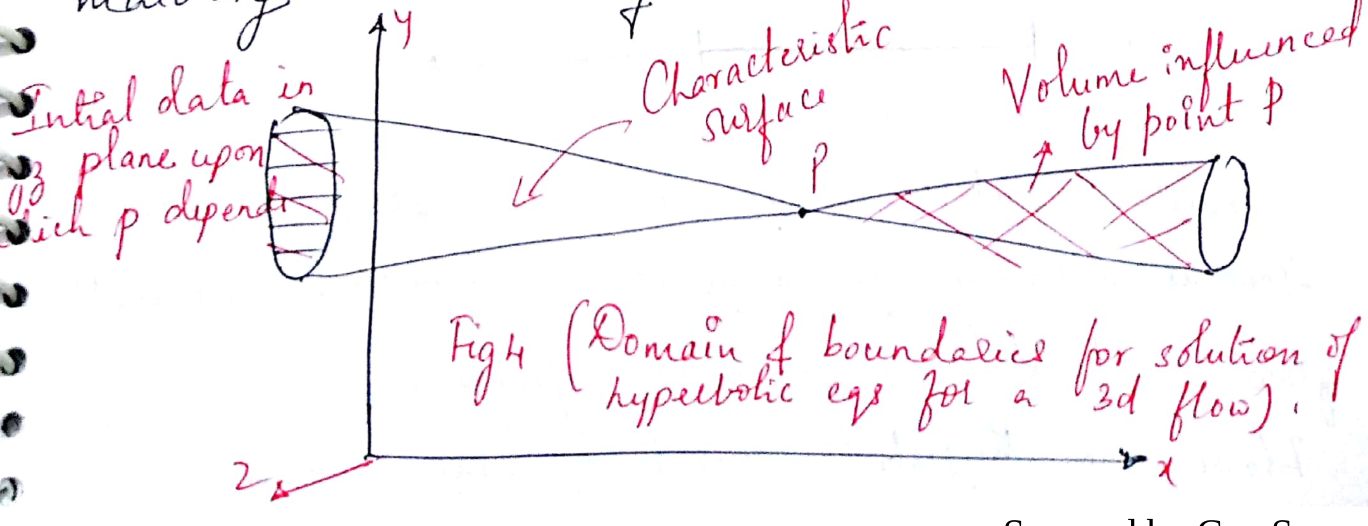

Fig 4 (Domain of boundaries for solution of hyperbolic eqs for a 3d flow).

For a 3d, supersonic inviscid flow as shown in fig 4 in 3d space xyz, the characteristics are surfaces. Consider point P, information at point P influences the shaded volume contained within the advancing characteristic surface.

In addition, if the yz plane is ad initial data surface, then only that portion of intial data shown as cross hatched area in xy plane, intercepted by retreating characteristic surface has any effect on P.

The dependent variables are solved by starting with data given in yz plane & marching in x-direction. For an inviscid flow problem the general flow direction will be also in x-direction.

Case 2:- <u>Unsteady, Inviscid flow</u>.

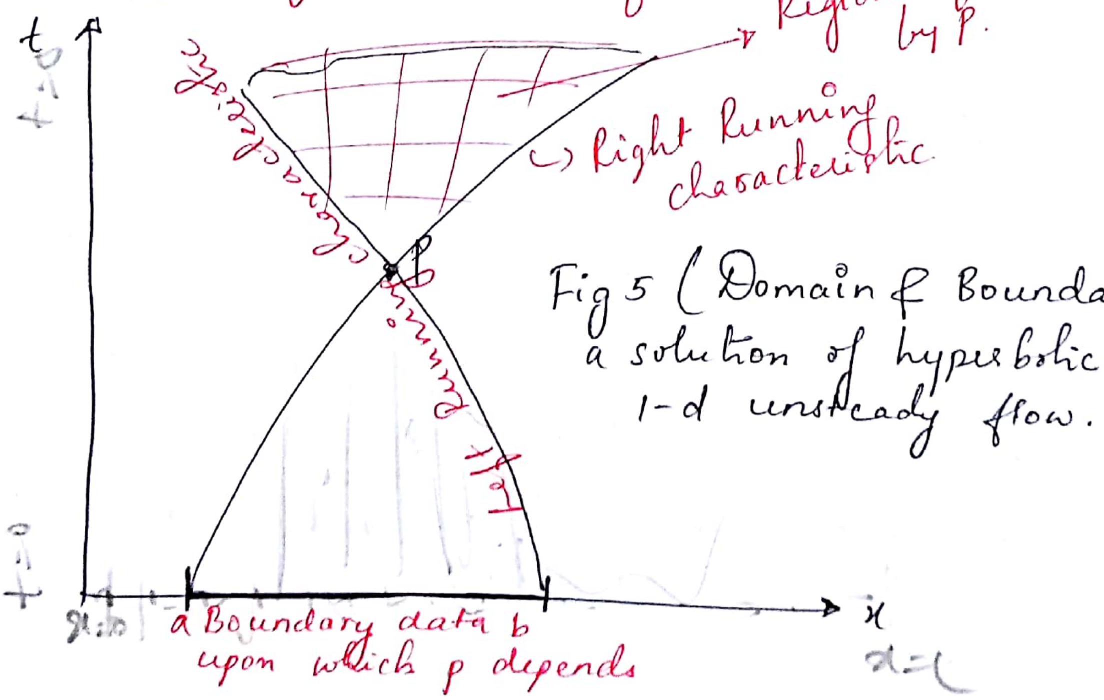

Fig 5 (Domain & Boundaries for a solution of hyperbolic eqs) 1-d unsteady flow.

Consider Euler's eqs. if time derivative are finite flow is unsteady, governing eqs are hyperbolic no matter flow is locally subsonic or supersonic.

Flows are hyperbolic w.r.t to time.

It ~~also implies whether~~ flow is ~~1d, or 2d or 3d~~,
In unsteady flows even if we have one, two or three spatial dimensions, the marching direction is time direction.

Consider a point as shown in fig 5 in xt plane.

~~Or~~. The region influed by P is shown by hatching b/w the two advancing characteristics through P.

The axis x(t=0) is intial data line. The interval ab is only portion on intial data along x-axis upon which the solution at P depends.

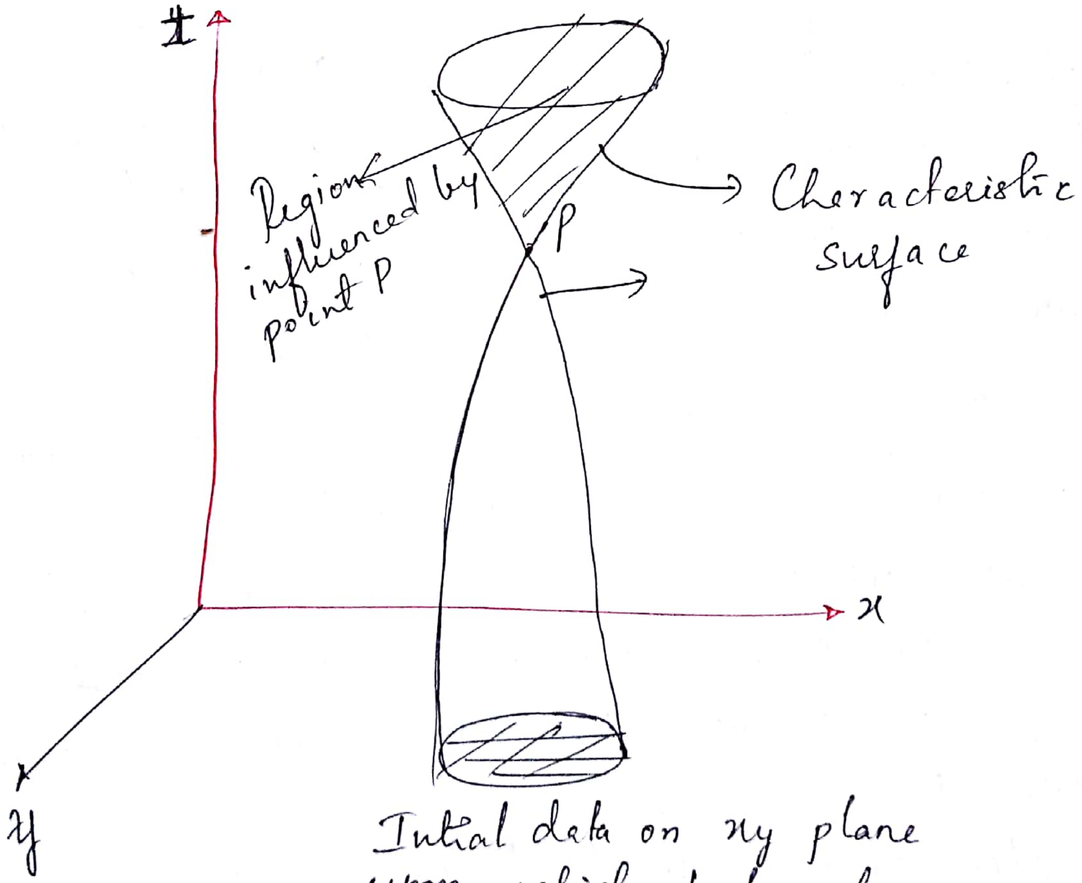

Fig 6;- (Domain & Boundaries for the solution of hyperbolic eqs - 2d unsteady flow)

For a 2d flow, consider a point in xyt plane as shown in fig 6. The region influenced by p & the portion of boundary on xy plane upon which the solution at P depends is given as shown in fig 6

Starting with intial data (known) in xy plane, the solution marches forward in time.

Parabolic Equations

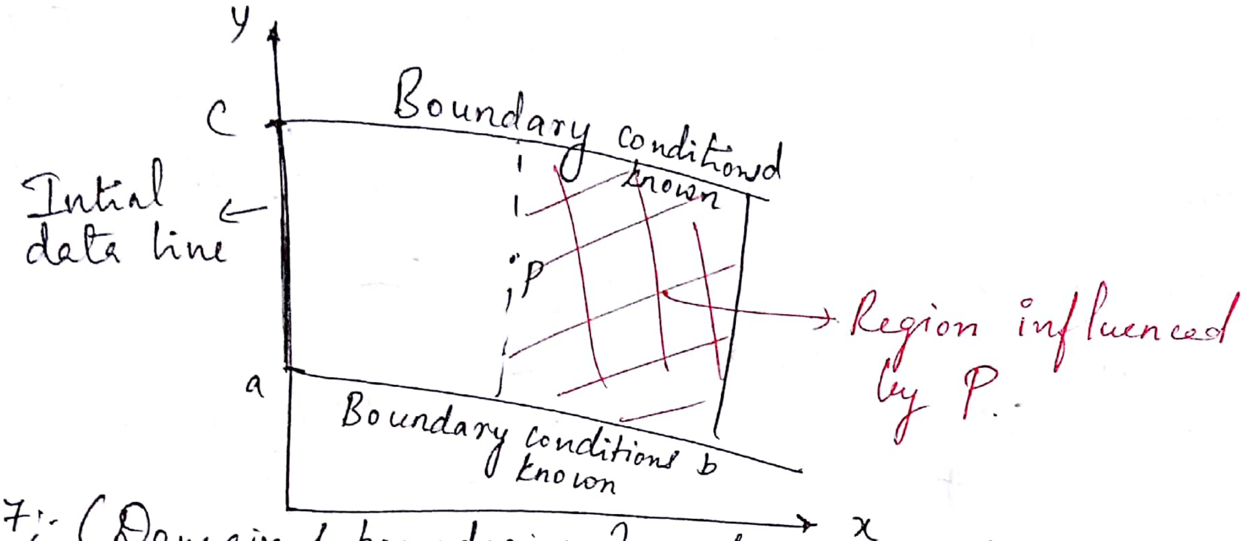

Fig 7:- (Domain & boundaries for soln of parabolic eqs in 2d)

Lets consider a parabolic eq in 2 dependent variables x & y. The xy plane is sketched as shown in fig. 7. Consider a point P given in this plane.

As we are dealing with parabolic equation, there is only one characteristic direction through point P.

Assume intial conditions are given along the line ac & boundary conditions are known along curves ab & cd. The characteristic direction is vertical line through P.

Then information at P influences the entire region on one side of P vertical characteristic & contained within 2 boundaries as shown in fig 8.

If we jab the needle at P, the effect of jab is felt throughout the hatched region shown in fig 8.7

Parabolic eqs like hyperbolic eqs lend themselves to marching soln. starting with intial data line ac, the solution b/w boundaries ab & cd is obtained by marching in x-direction.

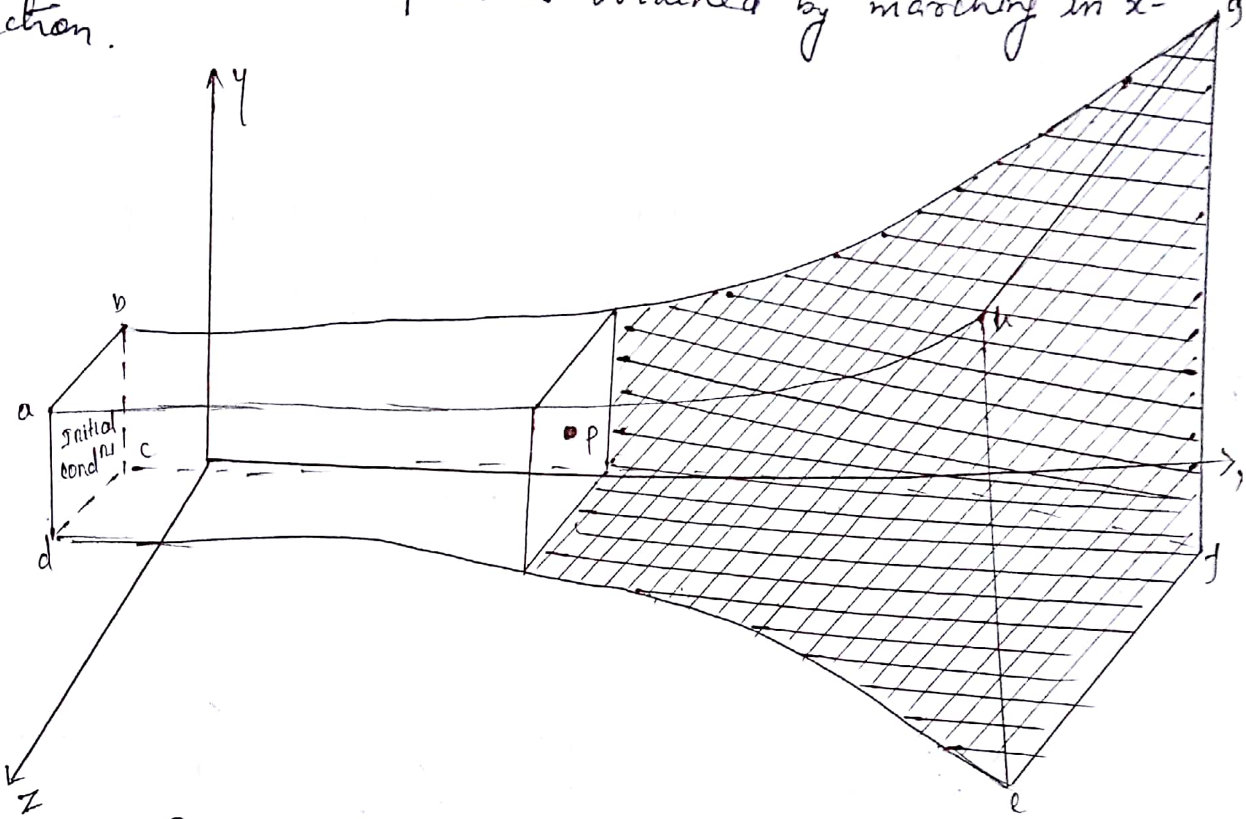

Fig 8 :- Domain & boundaries for solution of Parabolic eq in 3 dimension.

Here the parabolic eq has 3 independent variables x, y & z Consider point P located in this space. Assume intial conditions are given over area abcd in yz plane as shown in fig above. Assume boundary condition are given along four surfaces abgh, cdef, ahed & bgfc which extend in general x-direction away from the perimeter of intial data surface.

The information at P influences the whole 3d-region to the right of point P, contained within boundary surfaces. The region is shown by cross hatching in fig 8 Starting with intial data plane abcd, the solution is marched in x-direction.