Problem No- 4.2, 4.1. ($f(x) = \sin 2\pi x$) - Study them.

Reflection theory part pg :- No :- (137- 139)

Difference Equations :-

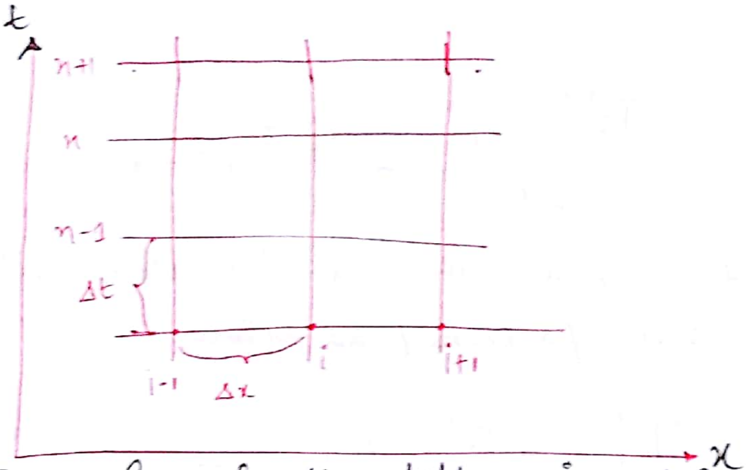

Fig 3:- Grid for the differencing of Eq.

When all the partial derivatives in a given a partial differential equations are replaced by finite difference quotients, the resulting algebraic eqs is called a difference equation, which is algebraic representation of PD eqs.

Lets consider ~~whi~~ a unsteady, 1 d heat conduction eqs with constant thermal diffusivity as given below.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \longrightarrow ①$$

① is parabolic partial differential eq., which leads itself to marching solution with respect to time t

From eq ① we have two independent variables x & t from the finite difference eqs. As shown in grid sketched above i is the running index in x- direction & n is running index in t direction. When one of the independent variables in partial differentiation eq in a marching variable such as t in eq ①, it is conventional in CFD to denote the running index for this marching variable by n & to display this index as a superscript in a finite difference quotient.

Let us replace time derivative with a forward difference as given below.

$$\left(\frac{\partial T}{\partial t}\right)_i^n = \frac{T_i^{n+1} - T_i^n}{\Delta t} - \left(\frac{\partial^2 T}{\partial t^2}\right)\frac{\Delta t}{2} + \cdots \rightarrow ②$$

Also let us replace the x-derivative with a central difference patterned as below.

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_i^n = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2} - \left(\frac{\partial^4 T}{\partial x^4}\right)_i^n \frac{(\Delta x)^2}{12} \rightarrow ③$$

Eq ① can be re-written as

$$\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = 0 \rightarrow ④$$

Substituting eq ② & eq ③ into eq. ④

$$\underbrace{\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = 0}_{\text{Partial Differential eqs}} = \underbrace{\left[\frac{T_i^{n+1} - T_i^n}{\Delta t} - \frac{\alpha\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right)}{(\Delta x)^2}\right]}_{\text{Difference equations}}$$

$$+ \underbrace{\left[-\left(\frac{\partial^2 T}{\partial t^2}\right)_i^n \frac{\Delta t}{2} + \alpha\left(\frac{\partial^4 T}{\partial x^4}\right)_i^n \frac{(\Delta x)^2}{12} + \cdots\right]}_{\text{Truncation error}} \longrightarrow ⑤$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{\alpha\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right)}{(\Delta x)^2} \longrightarrow ⑥$$

In eq. 5, the L.H.S is original P.D, first two terms on RHS are finite difference representation. & next 3rd & 4th terms in flower bracket are truncation terms.

Eq. 6 is difference eq which represents original P.D expressed by eq. 1, since each of finite different quotients used have their own error (truncation), the final form of the equation (difference) has its own truncation error synthesised from error of each final difference. Truncation error for Finite difference is $O[(\Delta t), (\Delta x)^2]$.

Difference eq is an algebraic eq. which when written at all grid points in a domain sketched in fig 4.3 yields a simultaneous system of algebraic eqs, which upon solving numerically yields dependent variable at each grid points

The only hope numerical results are similar in comparison to analytical solution is by having values of $\Delta x \to 0$, $\Delta t \to 0$ as they approach zero, difference eqs approaches to original differential eqs.

The numerical soln is approx representation of analytical soln from PD's if difference eqs are consistent numerical algorithms used to solve finite differences are stable & B.C's are handled properly.