SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Coimbatore – 641 035, Tamil Nadu

DEPARTMENT OF AEROSPACE ENGINEERING

19ASB304 Computational Fluid Dynamics for Aerospace

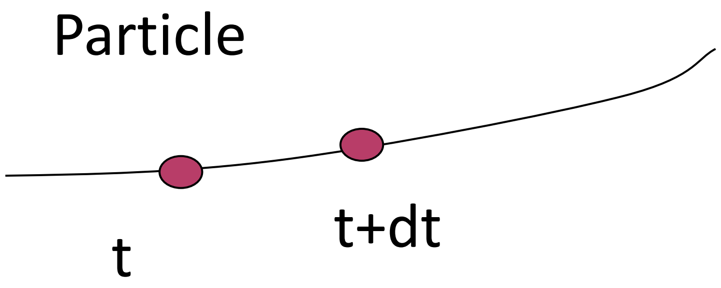
By

NEHRU.K

Assistant Professor

Aerospace Engineering

Particle Acceleration



Tracking the particle as we follow it path:

$$\vec{V}_{P@\text{time } t} = \vec{V}(x, y, z, t)$$

Note: V(x,y,z,t) is the velocity field of the entire flow, not the velocity of a particle.

As the particle moves, its velocity changes to

$$\vec{V}_{P@\text{time } t+dt} = \vec{V}(x + dx, y + dy, z + dz, t + dt)$$

The acceleration of a particle (substantial acceleration) is given by

$$\vec{a}_P = \frac{d\vec{V}_P}{dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{dx_P}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy_P}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz_P}{dt}$$

$$= \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}. \qquad \text{where } u = \frac{dx_P}{dt}, \quad v = \frac{dy_P}{dt}, \quad w = \frac{dz_P}{dt}$$

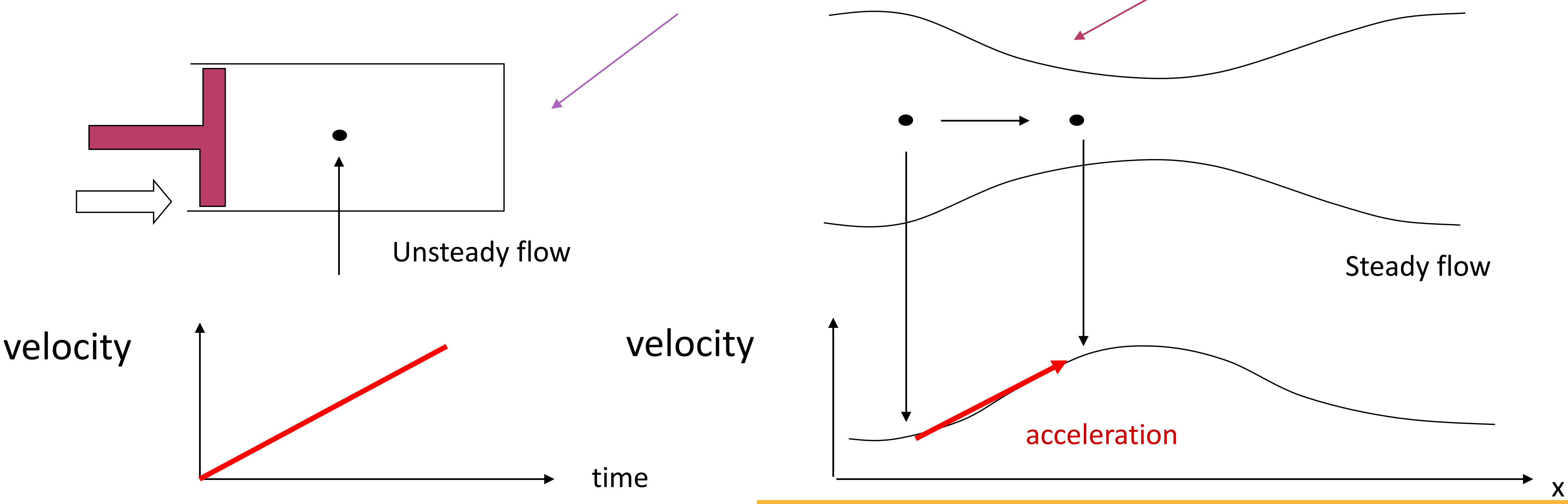
Physical Interpretation

$$\vec{a}_P = \boxed{\frac{D\vec{V}}{Dt}} = \boxed{\frac{\partial \vec{V}}{\partial t}} + \boxed{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}$$

Total acceleration
of a particle

Local
acceleration

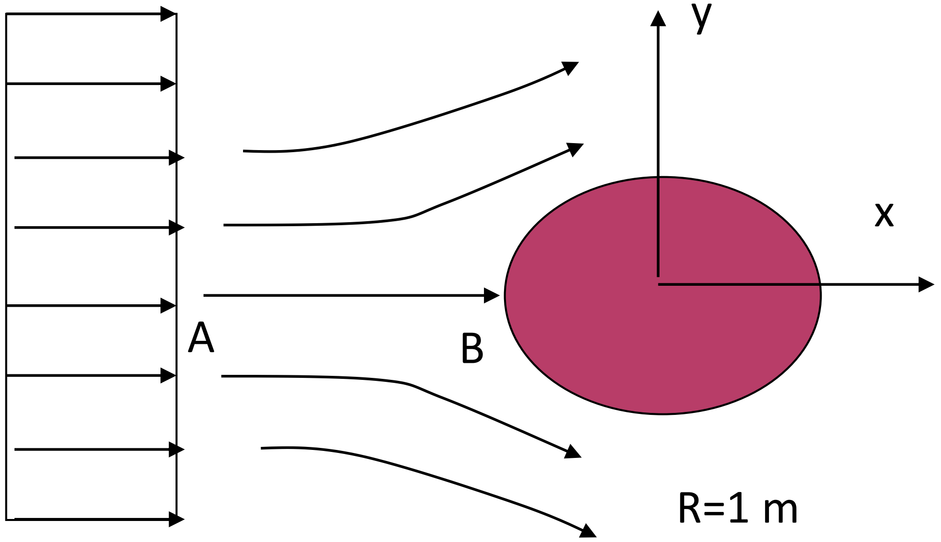
Convective acceleration

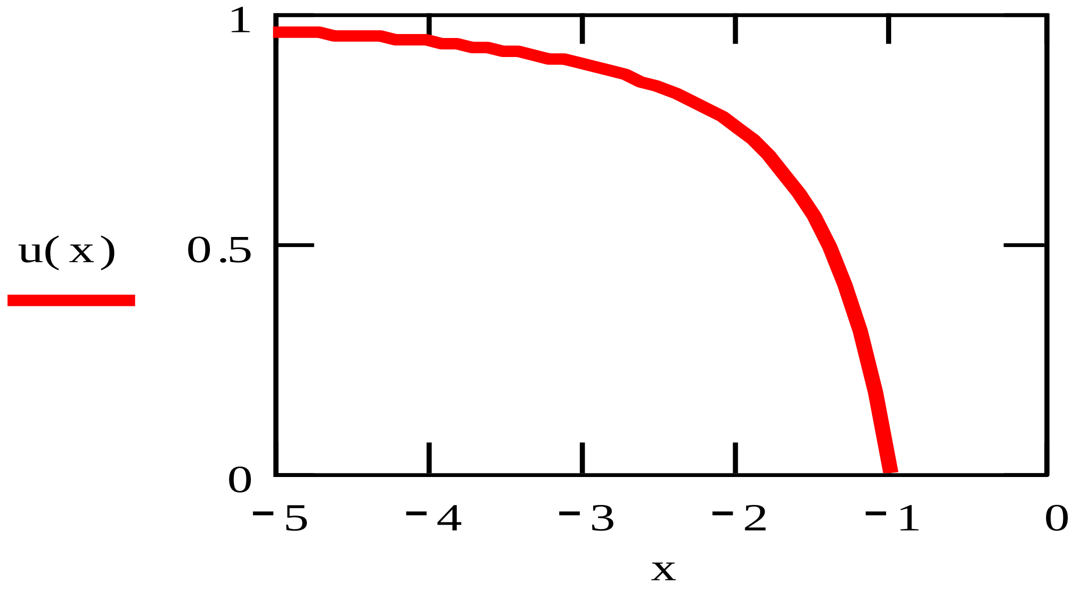


Example

An incompressible, inviscid flow past a circular cylinder of diameter d is shown below. The flow variation along the approaching stagnation streamline (A-B) can be expressed as:

$$\vec{V}(x, y=0) = u(x)\vec{i}, \qquad \text{where } u(x) = U_O\left(1 - \frac{R^2}{x^2}\right) = 1 - \frac{1}{x^2}$$





U_O=1 m/s

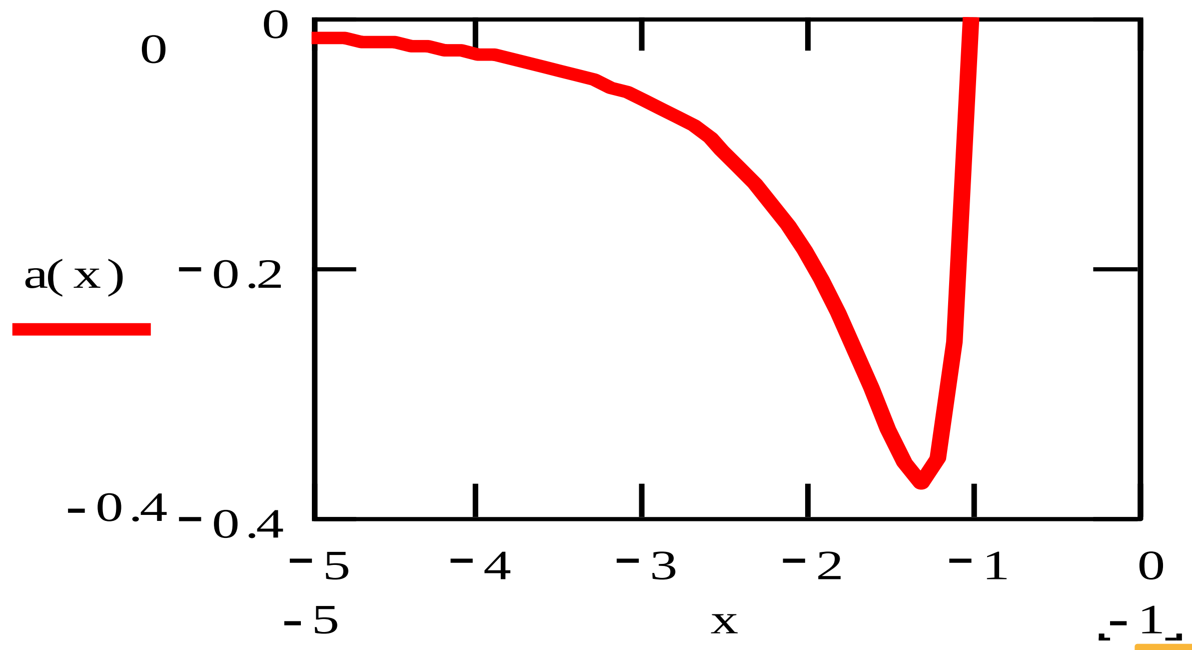
Along A-B streamline, the velocity drops very fast as the particle approaches the cylinder. At the surface of the cylinder, the velocity is zero (stagnation point) and the surface pressure is a maximum.

Example (cont.)

Determine the acceleration experienced by a particle as it flows along the stagnation streamline.

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + 0 + 0, \quad \text{since v = w = 0 along the stagnation streamline.}$$

$$\text{Therefore, } a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}, \quad a_y = a_z = 0, \quad a_x = (1 - \frac{1}{x^2})(\frac{2}{x^3}) \quad \text{for steady state flow}$$



- The particle slows down due to the strong deceleration as it approaches the cylinder.
- The maximum deceleration occurs at x=-1.29R=-1.29 m with a magnitude of a(max)=-0.372(m/s^2)

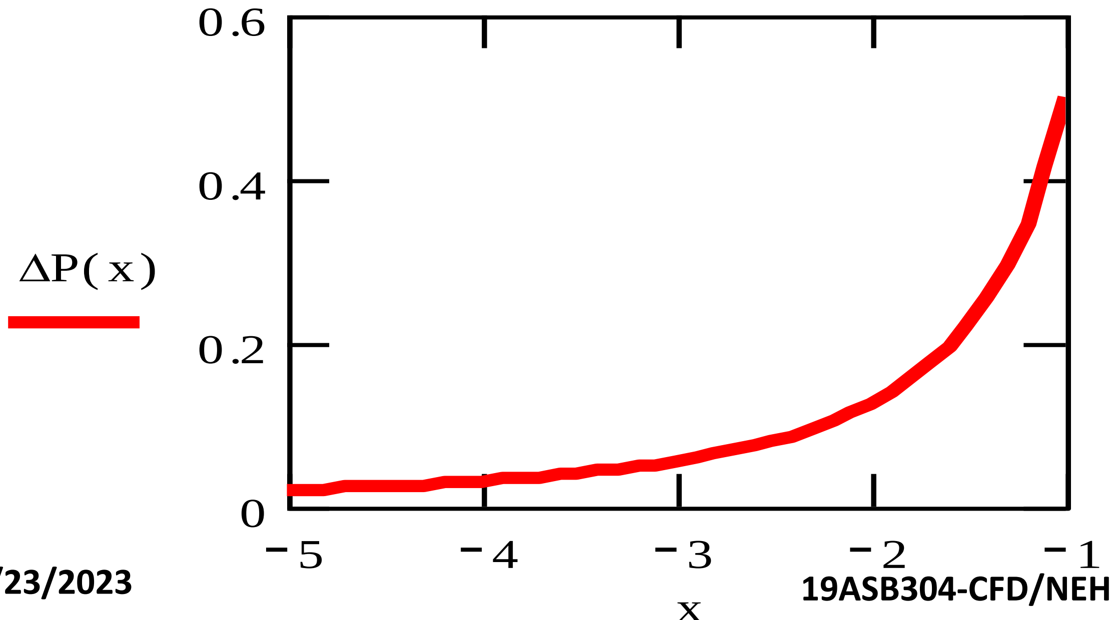
Example (cont.)

Determine the pressure distribution along the streamline using Bernoulli's equation. Also determine the stagnation pressure at the stagnation point.

$$\text{Bernoulli's equation:}\ \frac{\mathrm{P(x)}}{\rho}+\frac{\mathrm{u}^2(x)}{2}=\frac{P_\infty}{\rho}+\frac{U_O^2}{2}$$

$$P(x)-P_{atm}=\frac{\rho}{2}(U_O^2-u^2(x))=\frac{\rho}{2}\left(1-\left(1-\frac{1}{x^2}\right)\right)=\frac{\rho}{2}\left(\frac{1}{x^2}\right)$$

$$\Delta P(x)=\frac{P(x)-P_{atm}}{\rho}=\frac{1}{2x^2}$$

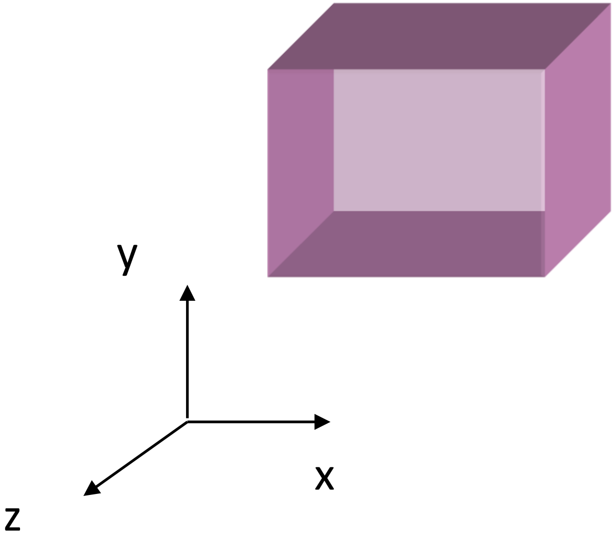


- The pressure increases as the particle approaches the stagnation point.
- It reaches the maximum value of 0.5, that is $P_{stag}-P_\infty=(1/2)\rho U_O^2$ as $u(x)\rightarrow 0$ near the stagnation point.

Momentum Conservation

From Newton's second law : Force = (mass)(acceleration)

Consider a small element $\delta x \delta y \delta z$ as shown below.



The element experiences an acceleration

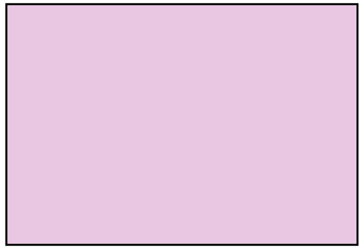
$$m\frac{D\vec{V}}{Dt} = \rho(\delta x \delta y \delta z)\left(\frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}\right)$$

as it is under the action of various forces:

normal stresses, shear stresses, and gravitational force

$$\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\delta y\right)\delta x \delta z$$

$$\sigma_{xx}\delta y \delta z$$



$$\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\delta x\right)\delta y \delta z$$

$$\tau_{yx}\delta x \delta z$$

Momentum Balance (cont.)

Net force acting along the x-direction:

$$\boxed{\frac{\partial \sigma_{xx}}{\partial x}\delta x\delta y\delta z} + \boxed{\frac{\partial \tau_{yx}}{\partial x}\delta x\delta y\delta z + \frac{\partial \tau_{zx}}{\partial x}\delta x\delta y\delta z} + \boxed{\rho g_x \delta x\delta y\delta z}$$

Normal stress

Shear stresses (note: τ_{zx}: shear stress acting on surfaces perpendicular to the z-axis, not shown in previous slide)

Body force

The differential momentum equation along the x-direction is

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{zx}}{\partial x} + \rho g_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

similar equations can be derived along the y & z directions

Euler's Equations

For an inviscid flow, the shear stresses are zero and the normal stresses are simply the pressure: $\tau = 0$ for all shear stresses, $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -P$

$$-\frac{\partial P}{\partial x} + \rho g_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

Similar equations for y & z directions can be derived

$$-\frac{\partial P}{\partial y} + \rho g_y = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$-\frac{\partial P}{\partial z} + \rho g_z = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

Note: Integration of the Euler's equations along a streamline will give rise to the Bernoulli's equation.

Navier and Stokes Equations

For a viscous flow, the relationships between the normal/shear stresses and the rate of deformation (velocity field variation) can be determined by making a simple assumption.
That is, the stresses are linearly related to the rate of deformation (Newtonian fluid).
The proportional constant for the relation is the dynamic viscosity of the fluid (μ).
Based on this, Navier and Stokes derived the famous Navier-Stokes equations:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$


Thank You