Unit - 1

Multiple integrals

Basics definitions and Formulas :

Integrals :

* The process of finding or solving definite and indefinite integrals are called integrals (or) The reciprocal of differentiation is called integrals.

Integration Formulas :

1. $\int 1 \, dx = x + c$
2. $\int a \, dx = ax + c$
3. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \, , \, n \neq -1$
4. $\int \sin x \, dx = -\cos x + c$
5. $\int \cos x \, dx = \sin x + c$
6. $\int \sec^2 x \, dx = \tan x + c$
7. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
8. $\int \sec x (\tan x) \, dx = \sec x + c$
9. $\int \operatorname{cosec} x (\cot x) \, dx = -\operatorname{cosec} x + c$
10. $\int \frac{1}{x} \, dx = \ln |x| + c$
11. $\int e^x \, dx = e^x + c = e^{2x} = \frac{e^{2x}}{2}$
12. $\int a^x \, dx = \frac{a^x}{\ln a} + c \quad a > 0, \ a \neq 1$
13. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$
14. $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$

16. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right)$

15. $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}x + C.$

Example : 1.

$\int (5x^2 - 8x + 5)\,dx.$

Soln :

Given,

$\int (5x^2 - 8x + 5)\,dx.$

$\Rightarrow \int [5x^2\,dx - 8x\,dx + 5\,dx]$

$= \int 5x^2\,dx - \int 8x\,dx + \int 5\,dx.$

$= 5\int x^2\,dx - 8\int x\,dx + 5\int dx.$

$= 5\left[\frac{x^3}{3}\right] - 8\left[\frac{x^2}{2}\right] + 5x.$

$= \frac{5x^3}{3} - 4x^2 + 5x$

Example : 2

$\int \left[\frac{8}{x} + \frac{5}{x^2} + \frac{6}{x^3}\right] dx$

Soln :

Given,

$\int \left[\frac{8}{x} + \frac{5}{x^2} + \frac{6}{x^3}\right] dx.$

$\Rightarrow \int \frac{8}{x}\,dx + \int \frac{5}{x^2}\,dx + \int \frac{6}{x^3}\,dx$

$= 8\int \frac{1}{x}\,dx + 5\int \frac{1}{x^2}\,dx + 6\int \frac{1}{x^3}\,dx.$

$= 8\int \frac{dx}{x} + 5\int x^{-2}\,dx + 6\int x^{-3}\,dx$

$= 8\log x + 5\left[\frac{x^{-1}}{-1}\right] + 6\left[\frac{x^{-2}}{-2}\right]$

$= 8\log x - \frac{5}{x} - \frac{3}{x^2}$

Example : 3

$\int 4e^{-7x} dx$

Soln:

Given,

$\int 4e^{-7x} dx$

$\Rightarrow 4\int e^{-7x} dx$

$= \frac{4e^{-7x}}{-7}$

$= -\frac{4e^{-7x}}{7}$

Example : 4

$\int_1^9 (x^{3/2} + 2x + 3)\, dx.$

Soln:

Given,

$\int_1^9 (x^{3/2} + 2x + 3)\, dx.$

$\Rightarrow \int_1^9 x^{3/2} dx + \int_1^9 2x\, dx + 3\int_1^9 dx.$

$= \left[\frac{x^{5/2}}{5/2}\right]_1^9 + 2\left[\frac{x^2}{2}\right]_1^9 + 3[x]_1^9$

$= \left[\frac{2x^{5/2}}{5}\right]_1^9 + 2\left[\frac{x^2}{2}\right]_1^9 + 3[x]_1^9$

$= \frac{2\times 9^{5/2}}{5}$

$= \frac{2\times\sqrt{9}\times 9^2}{5}$

$= \frac{2\times 3\times 81}{5}$

$= \frac{486}{5}$

$= \left[\left[\frac{2(9)^{5/2}}{5}\right] - \left[\frac{2(1)^{5/2}}{5}\right]\right] + 2\left[\frac{9^2}{2} - \frac{1^2}{2}\right] + 3[9-1]$

$= \left[\frac{486}{5} - \frac{2}{5}\right] + 2\left[\frac{81}{2} - \frac{1}{2}\right] + 3(8)$

$= \left[\frac{484}{5}\right] + \left[\frac{160}{2}\right] + 24$

$= 96.8 + 80 + 24$

$= 200.8$

$= 201$

Example : 5

$$\int (-6x^3 + 9x^2 + 4x - 3) \, dx$$

Soln :

Given ,

$$\int (-6x^3 + 9x^2 + 4x - 3) \, dx$$

$$\Rightarrow \int -6x^3 \, dx + \int 9x^2 \, dx + \int 4x \, dx - \int 3 \, dx$$

$$= -6 \int x^3 \, dx + 9 \int x^2 \, dx + 4 \int x \, dx - 3 \int dx$$

$$= -6 \left[\frac{x^4}{4}\right] + 9 \left[\frac{x^3}{3}\right] + 4 \left[\frac{x^2}{2}\right] - 3x$$

$$= -\frac{3x^4}{2} + 3x^3 + 2x^2 - 3x .$$