UNIT - II

Vector Calculus.

$\vec{i}, \vec{j}, \vec{k}$ - unit vectors

$\vec{a}.\vec{b}$ - Scalar product
$= \cos\theta$ (or)
dot product.

$\vec{a}\times\vec{b}$ - Vector product
$= ab\sin\theta$ (or)
cross product.

$\vec{a}\cdot(\vec{b}\times\vec{c}) = a\cdot(b\times c)$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- scalar triple product
(or)
box product.

Gradient (or Slope of a Scalar Point Function.

The vector differential operator ∇ is defined as

(1) $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\cdot\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

Definition: grad ϕ.

Let $\phi(x,y,z)$ be a real valued function having first order partial derivatives, Then $\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$.

$\nabla\phi$ is called the gradient of ϕ and is denoted by grad ϕ

$\therefore \boxed{\text{grad}\,\phi = \nabla\phi}$

Problems:

1) Find grad ϕ for $\phi = x^2+y^2+z^2$.

Solution:

Given: $\phi = x^2+y^2+z^2$

grad $\phi = \nabla\phi$

$= \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$.

$\frac{\partial\phi}{\partial x} = 2x$

$\frac{\partial\phi}{\partial y} = 2y$

$\frac{\partial\phi}{\partial z} = 2z$.

$\therefore$ grad ϕ

$= \vec{i}(2x) + \vec{j}(2y) + \vec{k}(2z)$

$\therefore$ grad ϕ

$= 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$.

2) Find grad ϕ where

$\phi = xyz$ at $(1,1,1)$

Solution:

Given $\phi = xyz$

grad $\phi = \nabla\phi$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}.$$

$$\frac{\partial\phi}{\partial x} = yz;\ \frac{\partial\phi}{\partial y} = xz;\ \frac{\partial\phi}{\partial z} = xy$$

$$\nabla\phi = \vec{i}(yz) + \vec{j}(xz) + \vec{k}(xy)$$

$$= yz\vec{i} + xz\vec{j} + xy\vec{k}.$$

$\nabla\phi$ at $(1,1,1)$

$$(\nabla\phi)_{(1,1,1)} = (1)(1)\vec{i} + (1)(1)\vec{j} + (1)(1)\vec{k}$$

$$\text{grad}\phi = \vec{i} + \vec{j} + \vec{k}.$$

3) Find grad ϕ where

$\phi = 3x^2y - y^3z^2$ at $(1,1,1)$

Solution:

Given $\phi = 3x^2y - y^3z^2$ at $(1,1,1)$

grad $\phi = \nabla\phi$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}.$$

$$\frac{\partial\phi}{\partial x} = 6xy;\ \frac{\partial\phi}{\partial y} = 3x^2 - 3y^2z^2$$

$$\frac{\partial\phi}{\partial z} = y^3 2z.$$

grad $\phi = \nabla\phi$

$$= \vec{i}(6xy) + \vec{j}(3x^2 - 3y^2z^2) + \vec{k}(2zy^3)$$

$$= 6xy\vec{i} + (3x^2 - 3y^2z^2)\vec{j} + 2zy^3\vec{k}.$$

$$(\nabla\phi)_{(1,1,1)}$$

$$= 6(1)(1)\vec{i} + (3(1)^2 - 3(1)(1))\vec{j} + 2(1)(1)^3\vec{k}$$

$$= 6\vec{i} + 0\vec{j} + 2\vec{k}.$$

$$\nabla\phi = 6\vec{i} + 2\vec{k}.$$

4) Find grad ϕ where

$\phi = x^2y^2z^2 + 4xz^2 + xy$ at $(1,2,3)$.

Soln:

Given:

$\phi = x^2y^2z^2 + 4xz^2 + xy$ at $(1,2,3)$

grad $\phi = \nabla\phi$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}.$$

$$\frac{\partial\phi}{\partial x} = (2x)y^2z^2 + 4z^2 + y$$

$$\frac{\partial\phi}{\partial y} = x^2z^2(2y) + 0 + x.$$

$$\frac{\partial \phi}{\partial z} = x^2y^2(2z) + 4x(2z) + 0$$

$$\nabla\phi = \vec{i}(2xy^2z^2 + 4z^2 + y)$$
$$+ \vec{j}(2x^2yz^2 + x)$$
$$+ \vec{k}(2x^2y^2z + 8xz)$$

$$(\nabla\phi)_{(1,2,3)}$$
$$= \vec{i}(2.1.2^2.3^2 + 4.3^2 + 2)$$
$$+ \vec{j}(2.1^2.2.3^2 + 1)$$
$$+ \vec{k}(2.1^2.2^2.3 + 8.1.3)$$

$$= \vec{i}(72 + 36 + 2)$$
$$+ \vec{j}(36+1) + \vec{k}(24+24)$$

$$= 110\vec{i} + 37\vec{j} + 48\vec{k}.$$

$$\therefore \text{grad}\,\phi = 110\vec{i} + 37\vec{j} + 48\vec{k}.$$

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