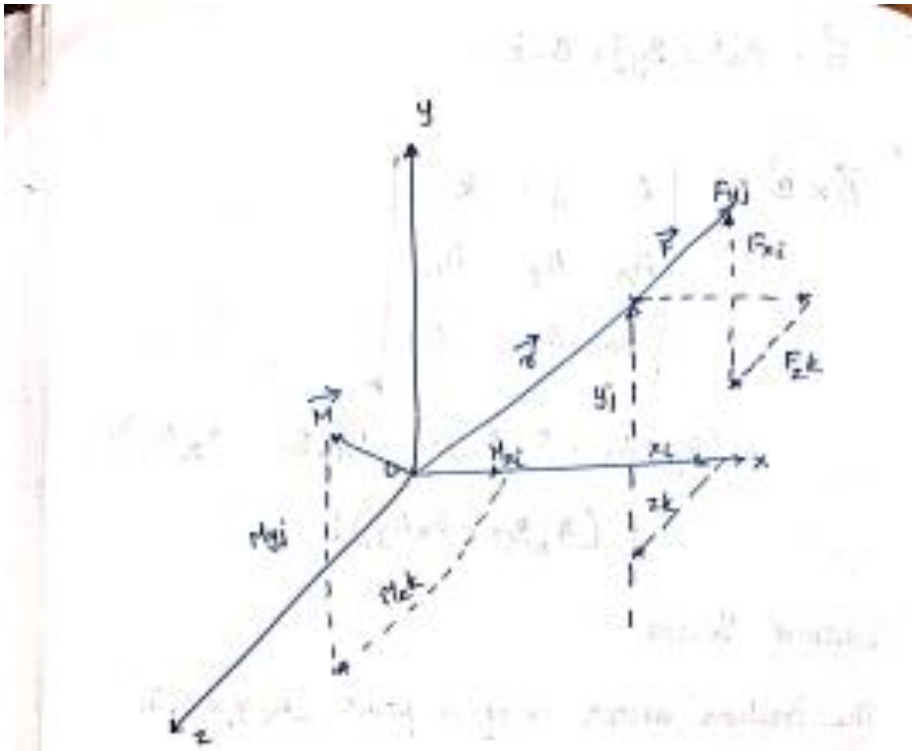


In fig,

$\vec{r}$ & $\vec{F}$ are in xy plane

moment of $\vec{F}$ about O is in zy plane ⊥ x

Let, $\vec{r} = xi + yj + zk$

$\vec{F} = F_x i + F_y j + F_z k$

$\vec{H} = H_x i + H_y j + H_z k$

$$\vec{H} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

The Scalar Quantity of $\vec{M}$ along the reference axes are obtained by dot product of $\vec{M}$ & unit vector along the respective axes.

Scalar of $\vec{M}$ along x axes $= \vec{M} \cdot i$

$= (M_x i + M_y j + M_z k) \cdot i$

$= M_x(1) + 0 + 0$

$= M_x$

Similarly M_y & M_z for y & z axes.

PROBLEMS

1. If $\vec{A} = 2i + 4j - 7k$

$\vec{B} = 3i - 5j + 6k$

Find (i) $\vec{A} + \vec{B}$

(ii) $\vec{A} - \vec{B}$

(iii) $\vec{A} \cdot \vec{B}$

(iv) $\vec{A} \times \vec{B}$

(i) $\vec{A} + \vec{B} = (2i + 4j - 7k) + (3i - 5j + 6k)$

$= 5i - j - k$

(ii) $\vec{A} - \vec{B} = (2i + 4j - 7k) - (3i - 5j + 6k)$

$= 2i + 4j - 7k - 3i + 5j - 6k$

$= -i + 9j - 13k$

(iii) $\vec{A} \cdot \vec{B} = (2i+4j-7k) \cdot (3i-5j+6k)$

$= (2\times3)+(4\times-5)+(-7\times6)$

$= 6-20-42$

$= -56$

(iv) $\vec{A} \times \vec{B} = (2i+4j-7k) \times (3i-5j+6k)$

$$= \begin{vmatrix} i & j & k \\ 2 & 4 & -7 \\ 3 & -5 & 6 \end{vmatrix}$$

$= i(24-35)-j(12+21)+k(-10-12)$

$= i(-11)-j(33)+k(-22)$

$= -11i-33j-22k$

2) If $\vec{P} = \overset{12i}{6}i + \overset{24}{12}j - \overset{10}{5}k$ & $\vec{Q} = \overset{9}{-3}i + \overset{10}{4}j - \overset{4}{2}k$

Find (i) $\frac{3}{4}\vec{P} + \frac{2}{3}\vec{Q}$

(ii) $\vec{P} \cdot \frac{4}{5}\vec{Q}$

(iii) $\frac{3}{2}\vec{P} \times \frac{4}{3}\vec{Q}$

(iv) $\left(\frac{2}{3}\vec{P} \times \vec{Q}\right) \cdot \left(\frac{1}{2}\vec{P} \times \frac{2}{4}\vec{Q}\right)$

(i) $4\vec{P} + 3\vec{Q}$

$4\vec{P} = 4(6i + 12j - 5k)$

$= 24i + 48j - 20k$

$3\vec{Q} = 3(-3i + 4j - 2k)$

$= -9i + 12j - 6k$

$4\vec{P} + 3\vec{Q} = (24i + 48j - 20k) + (-9i + 12j - 6k)$

$= 15i + 60j - 26k$

(ii) $\vec{P} \cdot 5\vec{Q}$

$5\vec{Q} = 5(-3i + 4j - 2k)$

$= -15i + 20j - 10k.$

$\vec{P} \cdot 5\vec{Q} = (6i + 12j - 5k) \cdot (-15i + 20j - 10k)$

$= [6 \times (-15)] + [12 \times 20] + [(-5) \times (-10)]$

$= -90 + 240 + 50$

$= 200.$

(iii) $2\vec{P} \times 3\vec{Q}$

$2\vec{P} = 2(6i + 12j - 5k)$

$= 12i + 24j - 10k$

$3\vec{Q} = 3(-3i + 4j - 2k)$

$= -9i + 12j - 6k$

$$\vec{2P} \times \vec{3Q} = \begin{vmatrix} i & j & k \\ 12 & 24 & -10 \\ -9 & 12 & -6 \end{vmatrix}$$

$= i[24 \times (-6)] - j[(12) \times (-6)]$

$= i[24 \times (-6) - (12 \times -10)] - j[(12 \times -6) - (-9 \times -10)]$
$+ k[(12 \times 12) - (-9 \times 24)]$

$= i[-144 + 120] - j[-70 - 90] + k[144 + 216]$

$= -24i + 162j + 360k.$

(iv) $(\vec{3P} \times \vec{Q}) \cdot (\vec{2P} \times \vec{4Q})$

(a) $\vec{3P} \times \vec{Q}$

$\vec{3P} = 3(6i + 12j - 5k)$

$= 18i + 36j - 15k$

$\vec{Q} = -3i + 4j - 2k$

$\therefore \vec{3P} \times \vec{Q} = \begin{vmatrix} i & j & k \\ 18 & 36 & -15 \\ -3 & 4 & -2 \end{vmatrix}$

$= i[(36 \times -2) - (4 \times -15)] - j[(18 \times -2) - (-3 \times -15)]$
$+ k[(18 \times 4) - (3 \times 36)]$

$= i[-72+60] - j[-36-45] + k[72+108]$

$= -12i + 81j + 180k$

(b) $2\vec{P} \times 4\vec{R}$

$2\vec{P} = 2(6i+12j-5k)$

$= 12i+24j-10k$

$4\vec{R} = 4(-3i+4j-2k)$

$= -12i+16j-8k$

$\therefore 2\vec{P} \times 4\vec{R} = \begin{vmatrix} i & j & k \\ 12 & 24 & -10 \\ -12 & 16 & -8 \end{vmatrix}$

$= i[(24\times-8)-(16\times-10)] - j[(12\times-8)-(-12\times-10)]$

$+ k[(16\times12)-(-12\times24)]$

$= i[-192+160] - j[-96-120] + k[192+288]$

$= -32i + 216j + 480k.$

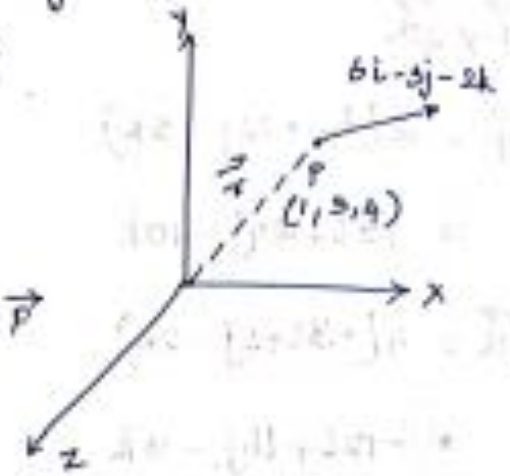
(c) $(3\vec{P} \times \vec{Q}) \cdot (2\vec{P} \times 4\vec{R})$

$= (-12i+81j+180k) \cdot (-32i+216j+480k)$

$= (-12\times-32) + (81\times216) + (180\times480)$

$= 104280$

3. A force $\vec{F} = 6i - 3j - 2k$ acts at a point P(1, 3, 4). Determine the moment of this force about the point of origin.



Coordinates of P(1, 3, 4)

∴ Position Vector,

$\vec{r} = i + 3j + 4k$

∴ Moment of the force $\vec{F}$ about O, $\vec{M}_o = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 6 & -3 & -2 \end{vmatrix}$$

$$= i(-6+12) - j(-2-24) + k(-3-18)$$

$$= 6i + 26j - 21k$$