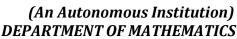


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Euler's method:  

$$y_{n+1} = y_n + h f(x_n, y_n)$$
,  $n = 0, 1, 2, ...$   
Modified Euler's method:  
 $y_{n+1} = y_n + \frac{1}{2} h \left[ f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right) \right]$   
Improved Euler's method:  
 $y_{n+1} = y_n + \frac{h}{2} \left[ f(x_n, y_n) + f(x_n + h, y_n + h(x_n, y_n)) \right]$   
Problems:  
() Using Euler's method find the solution of the initial  
value problem  $\frac{dy}{dx} = \log (x+y)$ ,  $y(0) = 2$  at  $x = 0.2$   
by assuming  $h = 0.2$ .  
Solution:  
Given:  $f(x, y) = \log (x+y)$   
 $x_0 = 0$ ,  $y_0 = 2$ ,  $x_1 = 0.2$ ,  $h = 0.2$   
 $y_{n+1} = y_n + h f(x_n, y_n)$   
For  $n = 0$ ,  $y_1 = y_0 + h f(x_0, y_0)$   
 $= 2 + (0.2) \log (n + 2)$   
 $= 2 + (0.2) \log (n + 2)$   
 $= 2 + (0.2) (0.3010)$   
 $y_1 = 2.0602$   
i.e.,  $y(0.2) = 2.0602$ 

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(2) Using Euler's method find y (0.2) and y (0.4) from  $\frac{dy}{dx} = x + y$ , y(0) = 1 with h = 0.2Solution : Given: f(x,y) = x+y $x_0 = 0$ ,  $y_0 = 1$ , h = 0.2 $x_1 = 0.2$ ,  $y_1 = ?$  $\chi_2 = 0.4$ ,  $y_2 = ?$ By Euler's formula,  $y_{n+1} = y_n + h f(x_n, y_n)$  $y_1 = y_0 + h f(x_0, y_0)$  $= 1 + (0.2) (\chi_{0} + Y_{0})$ = 1 + (0.2)(0+1)y(0.2) = 1.2 $y_2 = y_1 + h + f(x_1, y_1)$ = 1.2 + (0.2) (2, + y,)= 1.2 + (0.2) (0.2 + 1.2) $= 1 \cdot 2 + 0 \cdot 28$ y(0.4) = 1.48 3 Compute y at x = 0.25 by modified Euler method given y' = axy, y(0) = 1Solution : Given:  $x_0 = 0$ ,  $y_0 = 1$  $\chi_1 = 0.25, \ y_1 = ?$ h = 0.25



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(3)  

$$f(x, y) = axy$$
By modified Euler's method.  

$$y_{n+1} = y_n + h \left[ f\left(x_n + \frac{h}{a}, y_n + \frac{h}{a}, f(x_n, y_n)\right) \right]$$

$$y_1 = y_0 + h f\left(x_0 + \frac{h}{a}, y_0 + \frac{h}{a}, f(x_0, y_0)\right)$$

$$= 1 + (0.25) f\left(0 + \frac{0.25}{2}, 1 + \frac{0.25}{2}, f(0, 1)\right)$$

$$= 1 + (0.25) f\left(0.125, 1 + (0.125)(0)\right)$$

$$= 1 + (0.25) f\left(0.125, 1\right)$$

$$= 1 + (0.25) \left(2x \times 0.125 \times 1\right)$$

$$\frac{y(0.25) = 1.0625}{2}$$
By modified Euler's method, Compute  $y(0.1)$  with  

$$h = 0.1 \text{ from } y' = y - \frac{ax}{y}, y(0) = 1$$
Solution:  
Given:  $x_0 = 0, y_0 = 1, h = 0.1$   
 $x_1 = 0.1$   
 $f(x_1y) = y - \frac{2x}{y}$   
By modified Euler's method,  
 $y_{n+1} = y_n + h f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right)$   
 $y_1 = y_0 + h f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}f(x_0, y_0)\right)$ 

$$= 1 + (0.1) f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}f(0, 1)\right)$$

$$= 1 + (0.1) f\left(0.05 + 1.05\right)$$

$$= 1 + 0.1 (0.95 + 8.)$$

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