1) From the following table for bivariate distribution of (X, Y). Find

i) $P(X \le 1)$, ii) $P(Y \le 3)$, iii) $P(X \le 1, Y \le 3)$, iv) $P(X \le 1 / Y \le 3)$

v) $P(Y \le 3 / X \le 1)$ vi) Marginal distribution fn of X & Y

vii) Conditional distribution of X given Y = 2.

viii) Estimate X & Y are independent ; ix) $P(X + Y \le 4)$

X↓ Y→	1	2	3	4	5	6	P(X)
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{8}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{10}{16}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{8}{64}$
P(Y)	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	1

i) $P(X \le 1) = P(X=0) + P(X=1) = \frac{8}{32} + \frac{10}{16} = \frac{28}{32} = \frac{7}{8}$

ii) $P(Y \le 3) = P(Y=1) + P(Y=2) + P(Y=3) = \frac{3}{32} + \frac{3}{32} + \frac{11}{64} = \frac{23}{64}$

iii) $P(X \le 1, Y \le 3) = P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2)$

$X = 0, 1,\ Y = 1, 2, 3$

$+ P(1,3)$

$$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} = \frac{9}{32}$$

iv) $$P(X \le 1 / Y \le 3) = \frac{P(X \le 1, Y \le 3)}{P(Y \le 3)} = \frac{9/32}{23/64} = \frac{9}{\cancel{32}_{1}} \times \frac{\cancel{64}^{2}}{23} = \frac{18}{23}$$

v) $$P(Y \le 3 / X \le 1) = \frac{P(X \le 1, Y \le 3)}{P(X \le 1)} = \frac{9/32}{7/8} = \frac{9}{\cancel{32}_{4}} \times \frac{\cancel{8}^{1}}{7} = \frac{9}{28}$$

vi) Marginal distribution function of X :

X	0	1	2
P(X)	$\frac{8}{32}$	$\frac{10}{16}$	$\frac{8}{64}$

Marginal distribution function of Y:

Y	1	2	3	4	5	6
P(Y)	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$

vii) Conditional distribution function of X on Y = 2

$P(X \mid Y=2)$

$$P(X=0 / Y=2) = \frac{P(X=0, Y=2)}{P(Y=2)} = 0$$

$$P(X=1 / Y=2) = \frac{P(X=1, Y=2)}{P(Y=2)} = \frac{1/16}{3/32} = \frac{1}{16} \times \frac{32^{2}}{3} = \frac{2}{3}$$

$$P(X=2 / Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)} = \frac{1/32}{3/32} = \frac{1}{32} \times \frac{32}{3} = \frac{1}{3}$$

viii) X & Y are independent:

$\Rightarrow P(X=i, Y=j) = P(X=i) \cdot P(Y=j)$

Consider $P(2,3)$

$P(2,3) = P(X=2) \cdot P(Y=3)$

$$\frac{1}{64} \neq \frac{8}{64} \cdot \frac{11}{64}$$

$\therefore$ X and Y are not independent.

ix) $P(X+Y \le 4)$

$P(X+Y \le 4) = P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(1,1) + P(1,2) +$
$P(1,3) + P(2,1) + P(2,2)$

$$= 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32}$$

$$= \frac{1+2+2+2+4+1+1}{32} = \frac{13}{32}$$

2. If the joint p.d.f of (X, Y) is given by $P(X,Y) = k(2x+3y)$, $x = 0,1,2$; $y = 1,2,3$. Find all the marginal probability distribution. Also find the probability distribution of X+Y and $P(X+Y>3)$.

Given $P(x, y) = k(2x + 3y)$

X / Y	0	1	2	P(Y)
1	3k	5k	7k	15k
2	6k	8k	10k	24k
3	9k	11k	13k	33k
P(X)	18k	24k	30k	72k

Total probability $= 1 \Rightarrow 72k = 1 \quad \therefore k = \frac{1}{72}$

X / Y	0	1	2	P(x)
1	$\frac{3}{72}$	$\frac{5}{72}$	$\frac{7}{72}$	$\frac{15}{72}$
2	$\frac{6}{72}$	$\frac{8}{72}$	$\frac{10}{72}$	$\frac{24}{72}$
3	$\frac{9}{72}$	$\frac{11}{72}$	$\frac{13}{72}$	$\frac{33}{72}$
P(x)	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$	1

i) Marginal p. distribution function of X :

$P(X=0) = \frac{18}{72}$, $P(X=1) = \frac{24}{72}$, $P(X=2) = \frac{30}{72}$

Marginal probability distribution function of Y:

$P(Y=1) = \frac{15}{72}$, $P(Y=2) = \frac{24}{72}$, $P(Y=3) = \frac{33}{72}$

Probability distribution of X+Y:

ii) $P(X+Y=1) \qquad P(0,1) = \frac{3}{72}$

$P(X+Y=2) = P(1,1) + P(0,2) = \frac{5}{72} + \frac{6}{72} = \frac{11}{72}$

$P(X+Y=3) = P(2,1) + P(1,2) + P(0,3) = \frac{7}{72} + \frac{8}{72} + \frac{9}{72} = \frac{24}{72}$

$$P(X+Y=4) = P(2,2) + P(1,3) + \cancel{P(3,1)}$$

$$= \frac{10}{72} + \frac{11}{72} = \frac{21}{72}$$

$$P(X+Y=5) = P(2,3) = \frac{13}{72}$$

iii) $$P(X+Y>3) = P(X+Y=4) + P(X+Y=5)$$

$$= \frac{21}{72} + \frac{13}{72} = \frac{34}{72}$$