Chi-Square test for Independence of Attributes

$$\chi^2 = \Sigma \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

E_i = Expected frequency ; O_i = Observed frequency

$$E_i = \frac{(\text{Row total } B_i)(\text{column total } A_j)}{\text{Whole total}} \quad , \quad \begin{array}{l} i = 1 \text{ to } s \\ j = 1 \text{ to } t \end{array}$$

Degrees of freedom $\nu = (s-1 * t-1)$

1. On the basis of information noted below, find out whether the new treatment is comparatively superior to the conventional one.

	Favourable	Not favourable	Total
New	60	30	90
conventional	40	70	110
Total	100	100	200

Soln: To find E_i: $\frac{90 \times 100}{200} = 45$ $\frac{90 \times 100}{200} = 45$

$\frac{110 \times 100}{200} = 55$ $\frac{110 \times 100}{200} = 55$.

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
60	45	225	5
30	45	225	5
40	55	225	4.09
70	55	225	4.09
			18.18.

Null hypothesis:

H_0: There is no difference b/w new & conventional treatment

Alternative hypothesis.

H_1: There is a difference b/w new & conventional treatment

Level of significance: $\alpha = 5\%$ is fixed

Test statistics: $\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 18.18$

Degrees of freedom: $\gamma = (s-1) * (t-1) = (2-1) * (2-1)$

$= 1 * 1 = 1$

$\chi_\alpha^2 = 3.841$.

conclusion: $\chi^2 = 18.18 > 3.841 = \chi_\alpha^2$

H_0 is rejected at 5% level of significance.

i.e There is a difference between new & conventional treatment.

2. From the following table regarding the colour of eyes of father and son, test if the colour of son's eye is associated with that of the father.

		Eye colour of son	
		Light	Not light
Eye colour of father	Light	471	51
	Not light	148	230

To find E_i:

		Eye colour of son		
		Light	Not light	Total
Eye colour of father	Light	471	51	522
	Not light	148	230	378
Total		619	281	900

$E_{11} = \frac{522 \times 619}{900} = 359.02$ $\qquad E_{12} = \frac{522 \times 281}{900} = 162.98$

$E_{21} = \frac{378 \times 619}{900} = 259.98$ $\qquad E_{22} = \frac{378 \times 281}{900} = 118.02$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
471	359.02	111.98	12539.52	34.927
51	162.98	-111.98	12539.52	76.939
148	259.98	-111.98	12539.52	48.233
230	118.02	111.98	12539.52	106.249
				266.348

Null hypothesis: H_0: There is no ~~a~~ significant difference between the father & son's eye colour.

Alternative hypothesis: H_1: There is a difference b/w father & son's eye colour

Level of significance: $\alpha = 5\%$ is fixed

Test statistics: $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 266.348$

Degrees of freedom : $\nu = (r-1)(c-1) = (2-1)(2-1) = 1$

$\chi_\alpha^2 = 3.841$

Conclusion: $\chi^2 = 266.348 > 3.841 = \chi_\alpha^2$

$\therefore H_0$ is rejected at 5% level of significance.

(ii) There is a significant difference between father & son's eye colour.