

3. Height of father and son is given in cm.

x (height of father)	150	152	155	157	160	161	164	166
y (height of son)	154	156	158	159	160	162	161	164

Find a regression line or calculate expected average height of the son when the height of father is 154cm

$$\begin{array}{l}
 \text{x on y} \\
 (x - \bar{x}) = b_{xy} (y - \bar{y}) \\
 b_{xy} = r \frac{\sigma_x}{\sigma_y}
 \end{array}
 \quad \left| \quad
 \begin{array}{l}
 \text{y on x} \\
 (y - \bar{y}) = b_{yx} (x - \bar{x}) \\
 b_{yx} = r \frac{\sigma_y}{\sigma_x}
 \end{array}$$

$$r = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}, \quad \text{cov}(x, y) = E(xy) - E(x) \cdot E(y)$$

x	y	x <sup>2</sup>	y <sup>2</sup>	xy
150	154	22500	23716	23100
152	156	23104	24336	23712
155	158	24025	24964	24490
157	159	24649	25281	24963
160	160	25600	25600	25600
161	162	25921	26244	26082
164	161	26896	25921	26404
166	164	27556	26896	27224
1265	1274	200251	202958	201575

$$\bar{x} = \frac{\sum x}{n} = \frac{1265}{8} = 158.125, \quad \bar{y} = \frac{\sum y}{n} = \frac{1274}{8} = 159.25$$

$$\sigma_x = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{200251}{8} - (158.125)^2} = 5.2782$$

$$\sigma_y = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2} = \sqrt{\frac{202958}{8} - (159.25)^2} = 3.0311$$

$$r = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

$$\text{cov}(x, y) = \frac{\sum xy}{n} - \bar{x} \bar{y} = \frac{201575}{8} - (158.125)(159.25)$$

$$= 25196.875 - 25181.40625$$

$$= 15.46875$$

$$r = \frac{15.46875}{(5.2782)(3.0311)} = 0.9669$$

$$b_{xy} = r \frac{\sigma_x}{\sigma_y} = (0.9669) \frac{(5.2782)}{(3.0311)} = 1.6837$$

$$b_{yx} = r \frac{\sigma_y}{\sigma_x} = (0.9669) \frac{3.0311}{5.2782} = 0.5553$$

X on Y:

$$(x - \bar{x}) = b_{xy} (y - \bar{y}) \Rightarrow (x - 158.125) = 1.6837 (y - 159.25)$$

Y on X:

$$(y - \bar{y}) = b_{yx} (x - \bar{x}) \Rightarrow y - 159.250 = 0.5553 (x - 158.125)$$

Expected average height of the son when the height of the father is 154 cm.

$$(y - \bar{y}) = b_{yx} (x - \bar{x}) \Rightarrow (y - 159.250) = (0.5553) (x - 158.1250)$$

$$y = 0.5553 x - 86.9687 + 159.250$$

$$= 0.5553 (154) + 72.2813$$

$$= 156.9813$$

Average height of the son is 156.98 cm when the father height is 154 cm.