

3) The 2-D R.V. (X, Y) has joint p.m.f $f(x, y) = \frac{x+2y}{27}$,
 $x=0, 1, 2$; $y=0, 1, 2$. Find the conditional distribution
of Y for $X=x$. Also find conditional distribution of Y
given $x=1$.

Given $f(x, y) = \frac{x+2y}{27}$

$Y \backslash X$	0	1	2	$P(Y)$
0	0	$\frac{2}{27}$	$\frac{4}{27}$	$\frac{6}{27}$
1	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{5}{27}$	$\frac{9}{27}$
2	$\frac{2}{27}$	$\frac{4}{27}$	$\frac{6}{27}$	$\frac{12}{27}$
$P(X)$	$\frac{6}{27}$	$\frac{9}{27}$	$\frac{15}{27}$	1

i) $P(Y/x=x)$

When $x=0$:

$$P(Y=0/x=0) = \frac{P(x=0, y=0)}{P(x=0)} = \frac{0}{6/27} = 0$$

$$P(Y=1/x=0) = \frac{P(x=0, y=1)}{P(x=0)} = \frac{2/27}{6/27} = \frac{2}{6} = \frac{1}{3}$$

$$P(Y=2/x=0) = \frac{P(x=0, y=2)}{P(x=0)} = \frac{4/27}{6/27} = \frac{4}{6} = \frac{2}{3}$$

When $x=1$

$$P(Y=0/x=1) = \frac{P(x=1, y=0)}{P(x=1)} = \frac{1/27}{9/27} = \frac{1}{9}$$

$$P(Y=1/x=1) = \frac{P(x=1, y=1)}{P(x=1)} = \frac{3/27}{9/27} = \frac{3}{9} = \frac{1}{3}$$

$$P(Y=2/X=1) = \frac{P(X=1, Y=2)}{P(X=1)} = \frac{5/27}{9/27} = \frac{5}{9}$$

When $x=2$,

$$P(Y=0/X=2) = \frac{P(X=2, Y=0)}{P(X=2)} = \frac{2/27}{12/27} = \frac{2}{12} = \frac{1}{6}$$

$$P(Y=1/X=2) = \frac{P(X=2, Y=1)}{P(X=2)} = \frac{4/27}{12/27} = \frac{4}{12} = \frac{1}{3}$$

$$P(Y=2/X=2) = \frac{P(X=2, Y=2)}{P(X=2)} = \frac{6/27}{12/27} = \frac{6}{12} = \frac{1}{2}$$

ii) $P(Y/X=1)$

$$P(Y=0/X=1) = \frac{1}{9}$$

$$P(Y=1/X=1) = \frac{1}{3}$$

$$P(Y=2/X=1) = \frac{5}{9}$$