



# **SNS COLLEGE OF TECHNOLOGY**

**Coimbatore-35**  
**An Autonomous Institution**



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## **DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING**

### **19ECT211 – ELECTROMAGNETIC FIELDS**

II YEAR/ IV SEMESTER

#### **UNIT 5 – ELECTROMAGNETIC WAVES**

**TOPIC 3– WAVE EQUATION FOR A CONDUCTING MEDIUM & PLANE WAVES IN  
LOSSY DIELECTRICS**

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# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Maxwell's Equations for conducting medium

In a conducting medium with conductivity  $\sigma$  and charge density  $\rho$  and Maxwell's equations are as given below below.

<i>Differential form</i>	<i>Integral form</i>	<i>Eqn</i>
$\vec{\nabla} \cdot \vec{D} = \rho$	$\oint_S \vec{D} \cdot \vec{dS} = \int_V \rho_V dV$	<i>A</i>
$\vec{\nabla} \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot \vec{dS} = 0$	<i>B</i>
$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$	<i>C</i>
$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot \vec{dl} = \int_s \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \vec{dS}$	<i>D</i>



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Wave equations for lossy or conducting medium

- From equation (D)

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \frac{\partial}{\partial t} (\epsilon \vec{E})$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \text{ ---- (1)}$$

$$\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) = \sigma \frac{\partial \vec{E}}{\partial t} + \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} = \sigma \frac{\partial \vec{E}}{\partial t} + \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \text{ ---- (2)}$$



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Wave equations for lossy or conducting medium

- From equation (C)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

- Taking curl of this equation

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \left( \vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} \right) \text{ ---- (3)}$$

- Putting eq (2) in (3)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \left( \sigma \frac{\partial \vec{E}}{\partial t} + \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right) \text{ ---- (4)}$$

$$\text{But } \vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E} \text{ ---- (5)}$$



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Wave equations for lossy or conducting medium

- Using eq (5) in (4)

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \left( \sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right) \text{ ---- (4)}$$

$$\nabla^2 \vec{E} = \nabla(\nabla \cdot \vec{E}) + \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \text{ ---- (6)}$$

- But from equation (A) we have

$$\nabla \cdot \vec{D} = \rho \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

- Putting in equation (6)

$$\nabla^2 \vec{E} = \nabla \left( \frac{\rho}{\varepsilon} \right) + \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \text{ ---- (7)}$$



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Wave equations for lossy or conducting medium

- There is no charge within a conductor, although it may be there on the surface, the charge density  $\rho=0$ . So we can rewrite equation (7) as below.

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

- This is the wave equation for conducting medium in terms of E



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Wave equations for lossy or conducting medium

- From equation (D)

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \frac{\partial}{\partial t} (\epsilon \vec{E})$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{---- (1)}$$

- Taking the curl of this equation

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \sigma (\vec{\nabla} \times \vec{E}) + \epsilon \left( \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} \right) \quad \text{---- (2)}$$

$$\text{But } \vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} (\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H} \quad \text{---- (3)}$$



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



## Wave equations for lossy or conducting medium

- Using eq (3) in (2)

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H} = \sigma(\vec{\nabla} \times \vec{E}) + \epsilon \left( \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} \right) \quad \text{--- (4)}$$

- From equation (C)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (5)}$$

$$\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) = -\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} = -\frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{--- (6)}$$

- Putting in (5) and (6) in (4)

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H} = \sigma \left( -\frac{\partial \vec{B}}{\partial t} \right) + \epsilon \left( -\frac{\partial^2 \vec{B}}{\partial t^2} \right) \quad \text{--- (7)}$$





# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



Wave equations for lossy or conducting medium

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H} = -\mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{--- (8)}$$

$$\vec{\nabla}^2 \vec{H} = \vec{\nabla}(\vec{\nabla} \cdot \vec{H}) + \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{--- (9)}$$

$$\text{But } \vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot \vec{H} = 0$$

$$\vec{\nabla}^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\vec{\nabla}^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

- This is the wave equation for conducting medium in terms of H



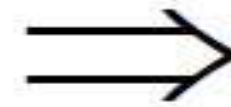
# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



Wave equations for lossy or conducting medium

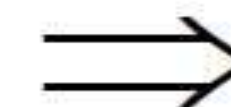
$$\sigma=0, \epsilon=\epsilon_0, \mu=\mu_0$$

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$



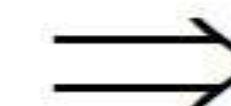
$$\nabla^2 \vec{E} = \mu_0\epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{D} = \mu\sigma \frac{\partial \vec{D}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{D}}{\partial t^2}$$



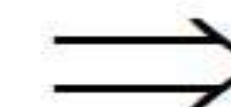
$$\nabla^2 \vec{D} = \mu_0\epsilon_0 \frac{\partial^2 \vec{D}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$



$$\nabla^2 \vec{H} = \mu_0\epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu\sigma \frac{\partial \vec{B}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$



$$\nabla^2 \vec{B} = \mu_0\epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$



# WAVE EQUATIONS FOR A CONDUCTING MEDIUM



Wave equations for lossy or conducting medium – time harmonic form

$$\vec{\nabla}^2 \vec{E} = j\omega\mu\sigma \vec{E} - \omega^2 \mu\epsilon \vec{E}$$

$$\vec{\nabla}^2 \vec{D} = j\omega\mu\sigma \vec{D} - \omega^2 \mu\epsilon \vec{D}$$

$$\vec{\nabla}^2 \vec{H} = j\omega\mu\sigma \vec{H} - \omega^2 \mu\epsilon \vec{H}$$

$$\vec{\nabla}^2 \vec{B} = j\omega\mu\sigma \vec{B} - \omega^2 \mu\epsilon \vec{B}$$



## REFERENCES

- John.D.Kraus , “ Electromagnetics “,5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: “Engineering Electromagnetics” Tata McGraw-Hill, 7th Edition 2007

THANK YOU