SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT211 – ELECTROMAGNETIC FIELDS

II YEAR/ IV SEMESTER

UNIT 5 – ELECTROMAGNETIC WAVES

TOPIC 3– WAVE EQUATION FOR A CONDUCTING MEDIUM & PLANE WAVES IN LOSSY DIELECTRICS

Maxwell's Equations for conducting medium

In a conducting medium with conductivity σ and charge density ρ and Maxwell's equations are as given below below.

Differential form	*Integral form*	*Eqn*
$\vec{\nabla}\cdot\vec{D}=\rho$	$\oint_S \vec{D}\cdot\vec{dS}=\int_V \rho_V dV$	*A*
$\vec{\nabla}\cdot\vec{B}=0$	$\oint_S \vec{B}\cdot\vec{dS}=0$	*B*
$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$	$\oint_L \vec{E}\cdot\vec{dl}=-\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{dS}$	*C*
$\vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$	$\oint_L \vec{H}\cdot\vec{dl}=\int_S\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{dS}$	*D*

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

- From equation (D)

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \frac{\partial}{\partial t}(\varepsilon \vec{E})$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad ---(1)$$

$$\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) = \sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} = \sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad ---(2)$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

- From equation (C)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

- Taking curl of this equation

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \left(\vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} \right) \quad ---(3)$$

- Putting eq (2) in (3)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \left(\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right) \quad ---(4)$$

$$But \quad \vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{E}\right) - \vec{\nabla}^2 \vec{E} \quad ---(5)$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

- Using eq (5) in (4)

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)-\vec{\nabla}^2\vec{E}=-\mu\left(\sigma\frac{\partial\vec{E}}{\partial t}+\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}\right)\quad ---(4)$$

$$\vec{\nabla}^2\vec{E}=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)+\mu\sigma\frac{\partial\vec{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}\quad ---(6)$$

- But from equation (A) we have

$$\vec{\nabla}\cdot\vec{D}=\rho \qquad \vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon}$$

- Putting in equation (6)

$$\vec{\nabla}^2\vec{E}=\vec{\nabla}\left(\frac{\rho}{\varepsilon}\right)+\mu\sigma\frac{\partial\vec{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}\quad ---(7)$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

- There is no charge within a conductor, although it may be there on the surface, the charge density ρ=0. So we can rewrite equation (7) as below.

$$\vec{\nabla}^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{\nabla}^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

- This is the wave equation for conducting medium in terms of E

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

- From equation (D)

$$\vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{H}=\sigma\vec{E}+\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)$$

$$\vec{\nabla}\times\vec{H}=\sigma\vec{E}+\varepsilon\frac{\partial\vec{E}}{\partial t}\quad ---(1)$$

- Taking the curl of this equation

$$\vec{\nabla}\times\vec{\nabla}\times\vec{H}=\sigma\left(\vec{\nabla}\times\vec{E}\right)+\varepsilon\left(\vec{\nabla}\times\frac{\partial\vec{E}}{\partial t}\right)\quad ---(2)$$

$$But\quad \vec{\nabla}\times\vec{\nabla}\times\vec{H}=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{H}\right)-\vec{\nabla}^2\vec{H}\quad ---(3)$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

- Using eq (3) in (2)

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{H}\right)-\vec{\nabla}^2\vec{H} = \sigma\left(\vec{\nabla}\times\vec{E}\right)+\varepsilon\left(\vec{\nabla}\times\frac{\partial\vec{E}}{\partial t}\right) \quad ---(4)$$

- From equation (C)

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad ---(5)$$

$$\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right) = -\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\vec{\nabla}\times\frac{\partial\vec{E}}{\partial t} = -\frac{\partial^2\vec{B}}{\partial t^2} \quad ---(6)$$

- Putting in (5) and (6) in (4)

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{H}\right)-\vec{\nabla}^2\vec{H} = \sigma\left(-\frac{\partial\vec{B}}{\partial t}\right)+\varepsilon\left(-\frac{\partial^2\vec{B}}{\partial t^2}\right) \quad ---(7)$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{H}\right)-\vec{\nabla}^2\vec{H} = -\mu\sigma\frac{\partial\vec{H}}{\partial t}-\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} \quad ---(8)$$

$$\vec{\nabla}^2\vec{H} = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{H}\right)+\mu\sigma\frac{\partial\vec{H}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} \quad ---(9)$$

$$But \quad \vec{\nabla}\cdot\vec{B}=0 \quad \Rightarrow \vec{\nabla}\cdot\vec{H}=0$$

$$\vec{\nabla}^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}$$

$$\boxed{\vec{\nabla}^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}}$$

- This is the wave equation for conducting medium in terms of H

Wave equations for lossy or conducting medium

$\sigma=0,\ \varepsilon=\varepsilon_0,\ \mu=\mu_0$

$$\vec{\nabla}^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} \Rightarrow \vec{\nabla}^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{\nabla}^2\vec{D} = \mu\sigma\frac{\partial\vec{D}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{D}}{\partial t^2} \Rightarrow \vec{\nabla}^2\vec{D} = \mu_0\varepsilon_0\frac{\partial^2\vec{D}}{\partial t^2}$$

$$\vec{\nabla}^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} \Rightarrow \vec{\nabla}^2\vec{H} = \mu_0\varepsilon_0\frac{\partial^2\vec{H}}{\partial t^2}$$

$$\vec{\nabla}^2\vec{B} = \mu\sigma\frac{\partial\vec{B}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} \Rightarrow \vec{\nabla}^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

WAVE EQUATIONS FOR A CONDUCTING MEDIUM

Wave equations for lossy or conducting medium – time harmonic form

$$\vec{\nabla}^2 \vec{E} = j\omega\mu\sigma \vec{E} - \omega^2 \mu\varepsilon \vec{E}$$

$$\vec{\nabla}^2 \vec{D} = j\omega\mu\sigma \vec{D} - \omega^2 \mu\varepsilon \vec{D}$$

$$\vec{\nabla}^2 \vec{H} = j\omega\mu\sigma \vec{H} - \omega^2 \mu\varepsilon \vec{H}$$

$$\vec{\nabla}^2 \vec{B} = j\omega\mu\sigma \vec{B} - \omega^2 \mu\varepsilon \vec{B}$$

REFERENCES

- John.D.Kraus , " Electromagnetics ",5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: "Engineering Electromagnetics" Tata McGraw-Hill, 7th Edition 2007

THANK YOU