SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT211 – ELECTROMAGNETIC FIELDS

II YEAR/ IV SEMESTER

UNIT 4 – TIME VARYING FIELDS & MAXWELL'S EQUATION

TOPIC 2 – MAXWELL'S EQUATION FOR STEADY AND TIME VARYING FIELDS

MAXWELL'S EQUATIONS

$$\nabla . E = \frac{\rho}{\epsilon_0}$$

$$\nabla . B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

FUNDAMENTAL RELATIONS

Along with Maxwell's equations certain other fundamental relations are of importance in dealing with electromagnetic problems. Among these may be mentioned Ohm's law at a point (4-9-4)

$$\mathbf{J} = \sigma \mathbf{E} \tag{14}$$

the continuity relation (4-13-3)

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \tag{15}$$

the force relations

$$\begin{aligned} \mathbf{F} &= q\mathbf{E} \\ d\mathbf{F} &= (\mathbf{I} \times \mathbf{B})\, dl \end{aligned} \tag{16}$$

MAXWELL'S EQUATION – FREE SPACE

In the preceding section, Maxwell's equations are stated in their general form. For the special case of free space, where the current density **J** and the charge density ρ are zero, the equations reduce to a simpler form. In integral form the equations are

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int_s \frac{\partial \mathbf{D}}{\partial t} \cdot ds \qquad (1)$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot ds \qquad (2)$$

$$\oint_s \mathbf{D} \cdot ds = 0 \qquad (3)$$

$$\oint_s \mathbf{B} \cdot ds = 0 \qquad (4)$$

MAXWELL'S EQUATION – FREE SPACE

In differential form the equations are

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (5)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (6)$$

$$\nabla \cdot \mathbf{D} = 0 \quad (7)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (8)$$

MAXWELL'S EQUATION – HARMONICALLY VARYING FIELDS

If we assume that the fields vary harmonically with time, Maxwell's equations can be expressed in another special form. Thus, if **D** varies with time as given by

$$\mathbf{D} = \mathbf{D}_0 e^{j\omega t} \qquad (1)$$

$$\frac{\partial \mathbf{D}}{\partial t} = j\omega \mathbf{D}_0 e^{j\omega t} = j\omega \mathbf{D}$$

MAXWELL'S EQUATION – HARMONICALLY VARYING FIELDS

When the same assumption is made for **B**, Maxwell's equations in integral form reduce to

$$\oint \mathbf{H} \cdot d\mathbf{l} = (\sigma + j\omega\epsilon) \int_s \mathbf{E} \cdot ds \qquad (3)$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -j\omega\mu \int_s \mathbf{H} \cdot ds \qquad (4)$$

$$\oint_s \mathbf{D} \cdot ds = \int_s \rho \, dv \qquad (5)$$

$$\oint_s \mathbf{B} \cdot ds = 0 \qquad (6)$$

MAXWELL'S EQUATION – HARMONICALLY VARYING FIELDS

In differential form they are

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} \quad (7)$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \quad (8)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (9)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (10)$$

STATIC & STEADY FIELDS

A static electric field (also referred to as electrostatic field) is created by charges that are fixed in space; A static magnetic field is created by a magnet or charges that move as a steady flow (as in appliances using direct current).

Steady fields does not vary with time.

MAXWELL'S EQUATION – INTEGRAL FORM TABLE

Table 9-1 MAXWELL'S EQUATIONS IN INTEGRAL FORM

Case \ Dimensions and SI units	From Ampère	From Faraday	From Gauss	
	mmf, A	emf, V	Electric flux, C	Magnetic flux, Wb
General	$F=\oint \mathbf{H}\cdot d\mathbf{l}=\int_s \left(\mathbf{J}+\frac{\partial \mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}=I_{total}$	$\mathcal{V}=\oint \mathbf{E}\cdot d\mathbf{l}=-\int_s \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$	$\psi=\oint_s \mathbf{D}\cdot d\mathbf{s}=\int_v \rho\, dv$	$\psi_m=\oint_s \mathbf{B}\cdot d\mathbf{s}=0$
Free space	$F=\oint \mathbf{H}\cdot d\mathbf{l}=\int_s \frac{\partial \mathbf{D}}{\partial t}\cdot d\mathbf{s}=I_{disp}$	$\mathcal{V}=\oint \mathbf{E}\cdot d\mathbf{l}=-\int_s \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$	$\psi=\oint_s \mathbf{D}\cdot d\mathbf{s}=0$	$\psi_m=\oint_s \mathbf{B}\cdot d\mathbf{s}=0$
Harmonic variation	$F=\oint \mathbf{H}\cdot d\mathbf{l}=(\sigma+j\omega\epsilon)\int_s \mathbf{E}\cdot d\mathbf{s}=I_{total}$	$\mathcal{V}=\oint \mathbf{E}\cdot d\mathbf{l}=-j\omega\mu\int_s \mathbf{H}\cdot d\mathbf{s}$	$\psi=\oint_s \mathbf{D}\cdot d\mathbf{s}=\int_v \rho\, dv$	$\psi_m=\oint_s \mathbf{B}\cdot d\mathbf{s}=0$
Steady	$F=\oint \mathbf{H}\cdot d\mathbf{l}=\int_s \mathbf{J}\cdot d\mathbf{s}=I_{cond}$	$V=\oint \mathbf{E}\cdot d\mathbf{l}=0$	$\psi=\oint_s \mathbf{D}\cdot d\mathbf{s}=\int_v \rho\, dv$	$\psi_m=\oint_s \mathbf{B}\cdot d\mathbf{s}=0$
Static	$U=\oint \mathbf{H}\cdot d\mathbf{l}=0$	$V=\oint \mathbf{E}\cdot d\mathbf{l}=0$	$\psi=\oint_s \mathbf{D}\cdot d\mathbf{s}=\int_v \rho\, dv$	$\psi_m=\oint_s \mathbf{B}\cdot d\mathbf{s}=0$

MAXWELL'S EQUATION – DIFFERENTIAL FORM TABLE

Table 9-2 MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM

	From Ampère	From Faraday	From Gauss	
Case \ Dimensions	Electric current / area	Electric potential / area	Electric flux / volume	Magnetic flux / volume
General	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{B} = 0$
Free space	$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \cdot \mathbf{D} = 0$	$\nabla \cdot \mathbf{B} = 0$
Harmonic variation	$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E}$	$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{B} = 0$
Steady	$\nabla \times \mathbf{H} = \mathbf{J}$	$\nabla \times \mathbf{E} = 0$	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{B} = 0$
Static	$\nabla \times \mathbf{H} = 0$	$\nabla \times \mathbf{E} = 0$	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{B} = 0$

REFERENCES

- John.D.Kraus , " Electromagnetics ",5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: "Engineering Electromagnetics" Tata McGraw-Hill, 7th Edition 2007

THANK YOU