1) find twist of cell shell of length 50cm of Aluminium Alloy as shown in fig. Take E = 70 GPa. Torque, T=100 KN

Soln:-

$E = 70 \times 10^9 \, Pa$

$E = 2G(1+\gamma)$

$\gamma = 0.33 \, (\text{for } Al)$

$70 \times 10^9 = 2G(1+0.33)$

$2.66 \, G = 70 \times 10^9$

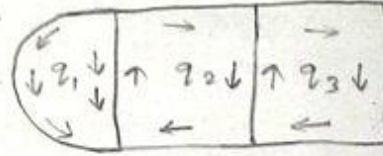$G = \dfrac{70 \times 10^9}{2.66}$

$G = 26.9 \, GPa$

Area of

Cell ① $A_1 = \dfrac{\pi r^2}{2} = 245.5 \, cm^2$

Cell ② $A_2 = l \times b = 25 \times 25 = 625 \, cm^2$

Cell ③ $A_3 = l \times b = 25 \times 25 = 625 \, cm^2$

$a_1 = \dfrac{S_1}{t_1} = \dfrac{2\pi r/2}{t_1} = \dfrac{\pi \times (12.5)}{0.1} = 392.7$

$a_{12} = \dfrac{S_{12}}{t_{12}} = \dfrac{25}{0.1} = 250$

$$a_2 = \frac{S_2}{t_2} = \frac{25}{0.2} + \frac{25}{0.2} = 250$$

$$a_{23} = \frac{S_{23}}{t_{23}} = \frac{25}{0.1} = 250$$

$$a_3 = \frac{S_3}{t_3} = \frac{25}{0.3} + \frac{25}{0.3} + \frac{25}{0.2} = 291.67$$

$$\theta = \frac{1}{2AG} \int q \frac{ds}{t}$$

for cell ①.

$$2G\theta_1 = \frac{1}{A_1}\left[q_1 a_1 + (q_1 - q_2) a_{12} \right]$$

$$= \frac{1}{245.7}\left[390.7 q_1 + 250 q_1 - 250 q_2 \right]$$

$$\boxed{2G\theta_1 = 2.618\, q_1 - 1.018\, q_2} \quad\text{———①}$$

$$2G\theta_2 = \frac{1}{A_2}\left[q_2 a_2 + (q_2 - q_1) a_{12} + (q_2 - q_3) a_{23} \right]$$

$$= \frac{1}{625}\left[250\, q_2 + 250 q_2 - 250 q_1 + 250 q_2 - 250 q_3 \right]$$

$$\boxed{2G\theta_2 = -0.4\, q_1 + 1.2\, q_2 - 0.4\, q_3} \quad\text{———②}$$

$$2G\theta_3 = \frac{1}{A_3}\left[q_3 a_3 + (q_3 - q_2) a_{23} \right]$$

$$= \frac{1}{625}\left[291.6 q_3 + 250 q_3 - 250 q_2 \right]$$

$$\boxed{2G\theta_3 = 0.866\, q_3 - 0.4\, q_2} \quad\text{———③}$$

$$T = 2A_1 q_1 + 2A_2 q_2 + 2A_3 q_3$$

$100 \times 10^3 \times 10^2 = 2 \times 245.7 \times q_1 + 2 \times 625 \times q_2 + 2 \times 625 \times q_3$

$$\boxed{10 \times 10^6 = 490.8\, q_1 + 1250\, q_2 + 1250\, q_3} \quad \underline{\qquad} \quad \text{④}$$

Assume $\theta_1 = \theta_2 = \theta_3 = 0$

$\text{①} = \text{②}$

$2.618\, q_1 - 1.018\, q_2 = -0.4\, q_1 + 1.2\, q_2 - 0.4\, q_3$

$2.618\, q_1 + 0.4\, q_1 - 1.018\, q_2 - 1.2\, q_2 + 0.4\, q_3 = 0$

$3.018\, q_1 - 2.218\, q_2 + 0.4\, q_3 = 0 \quad \underline{\qquad} \quad \text{⑤}$

$\text{②} = \text{③}$

$-0.4\, q_1 + 1.2\, q_2 - 0.4\, q_3 = 0.866\, q_3 - 0.4\, q_2$

$-0.4\, q_1 + 1.2\, q_2 + 0.4\, q_2 - 0.4\, q_3 - 0.866\, q_3 = 0$

$-0.4\, q_1 + 1.6\, q_2 - 1.266\, q_3 = 0 \quad \underline{\qquad} \quad \text{⑥}$

Solve ④, ⑤, ⑥

④ & ⑤,

$490.8\, q_1 + 1250\, q_2 + 1250\, q_3 = 10 \times 10^6 \quad \underline{\qquad}$