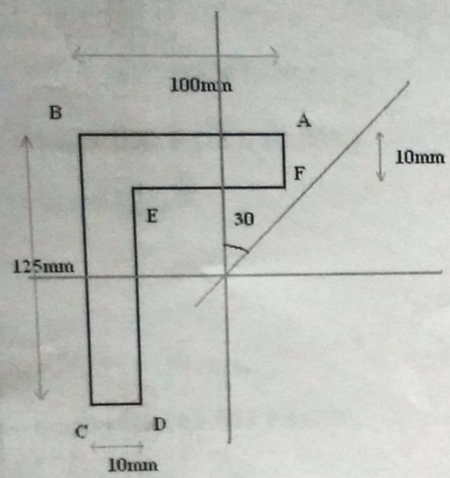
Subject Code & Name: **19AST203 Aircraft Structural Mechanics**

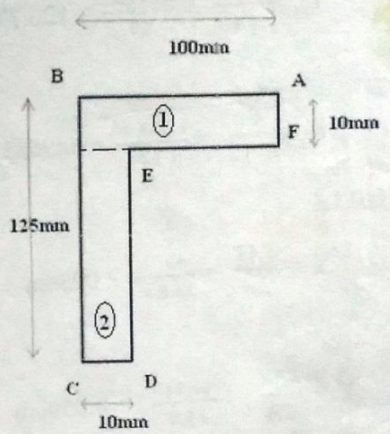
TOPIC: **12. Tutorial- Bending stresses**

Q.No .2:

A bending moment of 3000Nm is applied to the purlin in a plane at an angle 30 to vertical axis . if the sense of bending moment is such that component M_x and M_y both produces tension in positive xy quadrant .calculate the maximum direct stress in purlin stating clearly at which point it acts.

General Method (K Method):





Step : 1

Element table:

ELEMENT	A Cm^2	X cm	Y cm	AX	AY	AX^2	AY^2	AXY	I_{cx} Cm^4	I_{CY} Cm^4
1	10	4.5	12	45	120	202.5	1440	540	0.8333	83.33
2	11.5	0	5.75	0	66.125	0	380.2	0	126.73	0.9583
Σ	21.5			45	186.125	202.5	1820.2	540	127.5728	84.29

Section 1

$$I_{cx} = \frac{bd3}{12} = \frac{10X\ 1^3}{12} = 0.8333Cm^4$$

$$I_{cy} = \frac{db3}{12} = \frac{81X\ 100^3}{12} = 83.33\ Cm^4$$

Section 2

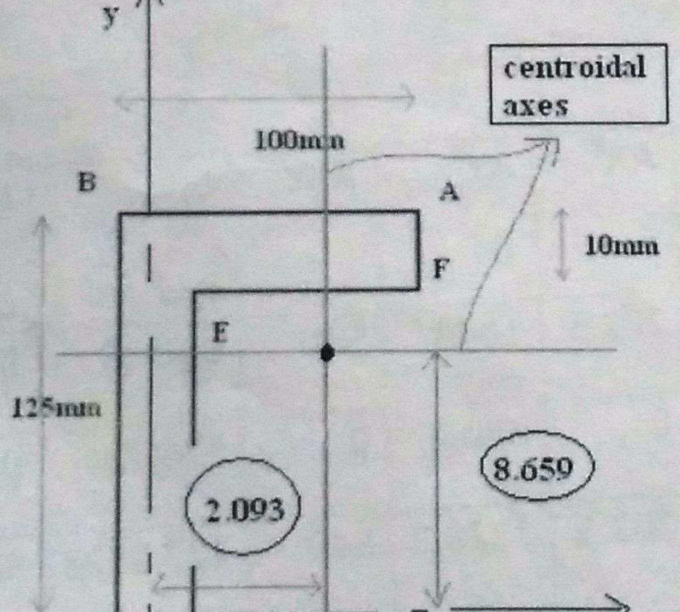
$$I_{cx} = \frac{bd3}{12} = \frac{1X\ 11.5^3}{12} = 126.73Cm^4$$

$$I_{cy} = \frac{db3}{12} = \frac{11.5X\ 1^3}{12} = 0.9583Cm^4$$

Step : 2

$$\bar{X} = \frac{\Sigma AX}{\Sigma A} = \frac{45}{21.5} = 2.093cm$$

$$\bar{Y} = \frac{\Sigma AY}{\Sigma A} = \frac{186.125}{21.5} = 8.659cm$$



Step : 3

$I_{xx} = \Sigma I_{cx} + \Sigma AY^2 - \Sigma a\,\overline{Y}^2$

$= 127.5728 + 1820.2 - (21.5 \times 8.659^2)$

$= 336.5347\ Cm^4$

$I_{yy} = \Sigma I_{cy} + \Sigma AX^2 - \Sigma a\,\overline{X}^{\,2}$

$= 84.29 + 202.5 - (21.5 \times 2.093^2)$

$= 192.607\ \ Cm^4$

$I_{xy} = \Sigma AXY - \Sigma A\overline{X}\overline{Y}$

$= 540 - (21.5 \times 2.093 \times 8.659)$

$= \quad 150.4438\ \ Cm^4$

Step : 4

$$\sigma = \frac{\overline{M}x}{Ixx} y + \frac{\overline{M}y}{Iyy} x$$

$M_x = M \sin 60 = 3000 \sin 60 = 1500\ Nm$

$M_y = M \cos 60 = 3000 \cos 60 = 2598.07\ Nm$

$$= \frac{0 - 1500\ X\ 10^{3}\ \frac{150.4438}{192.607}}{1 - \frac{150.4438^{\ 2}}{336.5347 X\ 192.607}}$$

$= 2191.80 \times 10^2$ N-cm

$$\overline{M}_y = \frac{My - Mx\frac{Ixy}{Ixx}}{1 - \frac{Ixy^{\ 2}}{IxxIyy}}$$

$$= \frac{1500\ X\ 10^{3} - 0\ X\ \frac{150.4438}{336.5347}}{1 - \frac{150.4438^{\ 2}}{336.5347 X\ 192.607}}$$

$=520 \times 10^2$ N- cm

Step : 5

$$\sigma\ = \frac{2191.80\ \text{x}10^{2}}{336.5347} y + \frac{520\text{x}10^{2}}{192.607} x$$

$\sigma\ = 6.512\ y + 2.70\ x$

	a	b	c	d	e	f
X	-2.593	7.407	7.407	-2.593	-1.593	-1.593
Y	3.843	3.843	2.843	-8.659	-8.657	2.843
$\sigma \times 10^2$ N/cm^2	18.024	45.024	38.512	-63.37	-60.675	14.21

Principal axis method

$I_{xx} = 336.5347\ Cm^4$

$I_{yy} = 192.607\ Cm^4$

$I_{xy} = 150.443\ Cm^4$

$M_x = 2598.076\ Nm = 2598.076 \times 10^2\ N\text{-}cm$

$M_y = 1500\ Nm = 1500 \times 10^2\ N\text{-}cm$

$$\tan 2\phi = \frac{-2\ Ixy}{Iyy - Ixx}$$

$\phi = -32^{o}.2'$

$I_{xx}^{p} = I_{xx}\cos^2\phi + I_{yy}\sin^2\phi - 2\ I_{xy}\sin\phi\cos\phi$

$I_{xx}^{p} = 431.33\ cm^4$

$I_{yy}^{p} = I_{xx}\sin^2\phi + I_{yy}\cos^2\phi + 2\ I_{xy}\sin\phi\cos\phi$

$I_{yy}^{p} = 97.8\ cm^4$

$M_x^{P} = M_x\cos\phi - M_y\sin\phi$

$M_x^{P} = 2997.768\ Nm$

$M_y^{P} = M_y\cos\phi + M_x\sin\phi$

$$\sigma^p = \frac{Mx^p}{I'_{xx}} y' + \frac{My^p}{I'_{yy}} x'$$

Y' = x cos ϕ + y sin ϕ

X' = y cos ϕ -x sin ϕ

$$\sigma^p = \frac{Mx^p}{I'_{xx}} (x \cos \varphi + y \sin \varphi) + \frac{My^p}{I'_{yy}} (y \cos \varphi - x \sin \varphi)$$

	a	b	c	d	e	f
X	-2.593	7.407	7.407	-2.593	-1.593	-1.593
Y	3.843	3.843	2.843	-8.659	-8.657	2.843
$\sigma \times 10^2$ N/cm^2	18.024	45.024	38.512	-63.37	-60.675	14.21

<u>**Neutral axis method**</u>

$I_{xx} = 336.5347\ Cm^4$

$I_{yy} = 192.607\ Cm^4$

$I_{xy} = 150.443\ Cm^4$

$M_x = 2598.076\ Nm = 2598.076 \times 10^2\ N\text{-}cm$

$M_y = 1500\ Nm = 1500 \times 10^2\ N\text{-}cm$

$I_N = I_{xx}\cos^2\alpha + I_{yy}\sin^2\alpha - 2\, I_{xy}\sin\alpha\ \cos\alpha$

$I_N = 208.94\ cm^4$

$M_N = M_x\cos\phi - M_y\sin\phi$

$M_N = 1825.28\ Nm$

$Y_N = x\cos\phi + y\sin\phi$

$= 0.923y - 0.383x$

$\tan\alpha = \frac{-\overline{M_y I_{xx}}}{\overline{M_x I_{yy}}}$

$\alpha = 22.52$

$\sigma = \frac{M_N}{I_N} y_N$

$= 8.73(0.923y - 0.383x)$

	a	b	c	d	e	f
X	-2.593	7.407	7.407	-2.593	-1.593	-1.593
Y	3.843	3.843	2.843	-8.659	-8.657	2.843
$\sigma \times 10^2$ N/cm^2	18.024	45.024	38.512	-63.37	-60.675	14.21