Euler's Method:

$$y_{n+1} = y_n + h f(x_n, y_n), \quad n = 0, 1, 2, \ldots$$

Modified Euler's method:

$$y_{n+1} = y_n + h \left[f\left(x_n + \frac{h}{2}, \ y_n + \frac{h}{2} f(x_n, y_n)\right) \right]$$

1] Using Euler's method, find the soln. of the initial value problem $\frac{dy}{dx} = \log(x+y)$, $y(0) = 2$ at $x = 0.2$ by assuming $h = 0.2$.

Soln.:

Given $f(x, y) = \log(x+y)$

$x_0 = 0 \quad y_0 = 2, \quad x_1 = 0.2, \quad h = x_1 - x_0 = 0.2$

$y_{n+1} = y_n + h f(x_n, y_n)$

$\therefore y_1 = y_0 + h f(x_0, y_0)$

$= 2 + (0.2) \log(x_0 + y_0)$

$= 2 + (0.2) \log(0 + 2)$

$= 2 + 0.2 \log 2$

$= 2 + (0.2)(0.3010)$

$y(0.2) = 2.0602$

2] Using Euler's method, find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x + y$, $y(0) = 1$ with $h = 0.2$

Soln.

Given $f(x, y) = x + y$

Here $x_0 = 0, \quad y_0 = 1, \quad h = 0.2$

$x_1 = 0.2, \quad y_1 = ?$

$x_2 = 0.4, \quad y_2 = ?$

By Euler's formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

$y_1 = y_0 + h f(x_0, y_0)$

$= 1 + (0.2)(x_0 + y_0)$

$$= 1 + (0.2)(0+1)$$
$$= 1 + 0.2$$
$$y(0.2) = 1.2$$

$$y_2 = y_1 + h\,f(x_1, y_1)$$
$$= 1.2 + (0.2)(x_1 + y_1)$$
$$= 1.2 + (0.2)(0.2 + 1.2)$$
$$= 1.2 + 0.28$$
$$y(0.4) = 1.48$$

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3. Using Euler's method, solve $y' = x + y + xy$, $y(0) = 1$. Compute y at $x = 0.1$ by taking $h = 0.05$

Soln.

Given $f(x, y) = x + y + xy$

Here $x_0 = 0$, $y_0 = 1$ and $h = 0.05$

$$y_1 = y_0 + h\,f(x_0, y_0)$$
$$= 1 + (0.05)(x_0 + y_0 + x_0 y_0)$$
$$= 1 + (0.05)(0 + 1 + 0)$$
$$y(0.05) = 1.05$$

$$y_2 = y_1 + h\,f(x_1, y_1)$$
$$= 1.05 + (0.05)[x_1 + y_1 + x_1 y_1]$$
$$= 1.05 + 0.05[0.05 + 1.05 + 0.05(1.05)]$$
$$= 1.05 + 0.05(1.1525)$$
$$= 1.05 + 0.0576$$
$$y_2 = 1.1076$$

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4. Compute y at $x = 0.25$ by modified Euler method given $y' = 2xy$, $y(0) = 1$

Soln.

Given $x_0 = 0$, $y_0 = 1$

$x_1 = 0.25$, $y_1 = ?$

$h = 0.25$

$f(x, y) = 2xy$

By modified Euler's method,

$y_{n+1} = y_n + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$

$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$

$= 1 + (0.25) f\left(0 + \frac{0.25}{2}, 1 + \frac{0.25}{2} f(0, 1)\right)$

$= 1 + (0.25) f(0.125, 1 + (0.125)(0))$

$= 1 + (0.25) f(0.125, 1)$

$= 1 + (0.25)(2 \times 0.125 \times 1)$

$y(0.25) = 1.0625$

5]. Using modified Euler's method, compute $y(0.1)$ with $h = 0.1$ from $y' = y - \frac{2x}{y}$, $y(0) = 1$

Soln.

Given $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$x_1 = 0.1$, $y_1 = ?$

$f(x, y) = y - \frac{2x}{y}$

By modified Euler's method,

$y_{n+1} = y_n + h\, f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)$

$y_1 = y_0 + h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)$

$= 1 + (0.1) f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2} f(0, 1)\right)$

$= 1 + 0.1\, f(0.05, 1.05)$

$= 1 + 0.1\left(1.05 - \frac{2(0.05)}{1.05}\right)$

$= 1 + 0.1\,(0.9548)$

$y(0.1) = 1.09548$

Hw 1. Using modified Euler's method, find $y(0.1)$ if $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$

2. Use Euler's method to find $y(0.4)$ given $y' = xy$, $y(0) = 1$