Test of significance of small samples:

The techniques examined in previous section are based only on large samples $(N \geq 30)$

If the sample size is small $(N < 30)$ Then the sampling distribution of many statistics are not normal and we have to develop entirely different test of significance based on t - Test (mean), F - test (Variance) & χ^2 (chi) distribution.

Degrees of freedom:

Number of degrees of freedom is the total number of observation − Number of independent constrained imposed on the observation

(ie) n is no. of observation

k is no. of independent constraint then

$n-k$ is the degrees of freedom.

The No. of degrees of freedom is usually denoted by ν (Nu)

Test of hypothesis about population mean (students - t - distribution)

Under this type the test statistic is given by

$$t = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n}}\right)} \sim t_{(n-1)} \quad \text{Where}$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

1) A random sample of 10 boys have the following IQ: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do this data support the assumption of a population mean IQ of 100? Find a reasonable range in which most of mean IQ values of samples of 10 boys lie.

Sol:

Here $n = 10$ (small sample)

Gn: $\mu = 100$

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$ (Two-tailed test)

Now $\bar{x} = \dfrac{70+120+110+101+88+83+95+98+107+100}{10}$

$\bar{x} = 97.2$

x	$x_i - \bar{x} = x_i - 97.2$	$(x_i - \bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84

$\Sigma(x_i - \bar{x})^2 = 1833.6$.

1833.6

W.K.T.

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

$$s^2 = \frac{1}{9} [1833.6]$$

$$s^2 = 203.73$$

$$s = 14.27$$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} \sim t_{n-1}$$

$$t = \frac{97.2 - 100}{\left(\frac{14.27}{\sqrt{10}}\right)}$$

$$t = -0.62$$

$$|t| = 0.62$$

Let $\alpha = 0.05$

The critical value of t for two tailed test at 5 percent level of significance with degrees of freedom '9' is 2.262

$$|t| = 0.62 < 2.262$$

$\therefore$ We accept our null hypothesis.

$\therefore$ We conclude that the data support the population mean IQ of 100.

Reasonable range

$$95\%\ \text{C.L} = \left(\bar{x} - t_{0.05}\left(\frac{s}{\sqrt{n}}\right),\ \bar{x} + t_{0.05}\left(\frac{s}{\sqrt{n}}\right)\right)$$

$$= \left(97.2 - 2.262\left(\frac{14.27}{\sqrt{10}}\right),\ 97.2 + 2.262\left(\frac{14.27}{\sqrt{10}}\right)\right)$$

$$= (86.9;\ 107.4)$$