TEST FOR DIFFERENCE FOR TWO MEANS :

Null hypothesis : $H_0 : \mu_1 = \mu_2$

test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ $\quad \sigma_1 = \sigma_2 = \sigma$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

(or) $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

1) The means of two simple large samples of 1000 and 2000 members are 67.5 inches and 68 inches resp. Can the samples be regarded as drawn from the same population of standard deviation of 2.5 inches? Test at 5% level of significance (LOS)

Soln:

Given : $n_1 = 1000$, $\bar{x}_1 = 67.5$,
$n_2 = 2000$, $\bar{x}_2 = 68$, & $\sigma = 2.5$

step 1: Formulating H_0 and H_1 :

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two tailed test)

Step 2 : Level of significance, $\alpha = 5\% = 0.05$

Step 3 : Test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$= \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}}$$

$$= -5.164$$

$$|z| = |-5.164|$$

$$= 5.164$$

Step 4 : critical value, at 5% (two sided test) is $z_\alpha = 1.96$.

Step 5 : Conclusion ; $z = 5.164 > 1.96 = z_\alpha$

$\therefore$ Ho is rejected at 5% LOS.

$\therefore$ The samples cannot be regarded as drawn from the same population of S.D. 2.5 inches.

A simple sample of height of 6400 sailors has a mean of 67.85 inches and S.D. of 2.56 inches while a simple sample of heights of 1600 soldiers has a mean of 68.55 inches and S.D. of 2.52 inches. Do the data, indicate that soldiers are on the average taller than sailors ? use 5% LOS.

Soln:

Given: Sailors: $n_1 = 6400$, $\bar{x}_1 = 67.85$, $s_1 = 2.56$

Soldiers: $n_2 = 1600$, $\bar{x}_2 = 68.55$, $s_2 = 2.52$.

Step 1: Formulating H_0 and H_1

$H_0 : \mu_1 = \mu_2$.

$H_1 : \mu_1 < \mu_2$ (one tailed test - left)

Step 2: LOS at 5% (or) $\alpha = 0.05$

Step 3: Test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$$= \frac{67.85 - 68.55}{\sqrt{\frac{(2.56)^2}{6400} + \frac{(2.52)^2}{1600}}}$$

$$= -9.91$$

$$|z| = |-9.91|$$

$$= 9.91$$

Step 4: Critical value at 5% (one tail test)

is $z_\alpha = 1.645$

Step 5: Conclusion: $z = 9.91 > 1.645 = z_\alpha$

$\therefore H_0$ is rejected at 5% of LOS

$\therefore$ The data indicates that soldiers are on the average taller than sailors.

* A simple sample of heights of 6400 English men has a mean of 170 cm & S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm & S.D. of 6.3 cm. Do the data indicate that Americans are the avg. taller than the english men? [$z = 11.32$, $\mu_1 < \mu_2$, Americans are taller than English men]