# Interpolation with Equal Intervals

## Newton's forward Interpolation Formula:

Let $y = f(x)$ be a function. Then

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots\dots$$

where $u = \frac{x - x_0}{h}$ and $h$ is the difference.

## Newton's Backward Interpolation Formula:

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \dots$$

where $v = \frac{x - x_n}{h}$ and $h$ is the difference

1. The population of a town is as follows

| Year | 1941 | 1951 | 1961 | 1971 | 1981 | 1991 |
|---|---|---|---|---|---|---|
| population in lakhs | 20 | 24 | 29 | 36 | 46 | 51 |

Estimate the population increase during the period 1946 to 1976.

Soln.

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 1941 | 20 | | | | | |
| | | 4 | | | | |
| | | | 1 | | | |
| 1951 | 24 | | | 1 | | |
| | | 5 | | | 0 | |
| 1961 | 29 | | 2 | | | |
| | | | | 1 | | −9 |
| | | 7 | | | | |
| 1971 | 36 | | 3 | | −9 | |
| | | 10 | | −8 | | |
| 1981 | 46 | | −5 | | | |
| | | 5 | | | | |
| 1991 | 51 | | | | | |

Here $h = 10$

Since $x = 1946$ is nearer to the beginning of the table, we use Newton's forward difference table formula.

Here $u = \frac{x - x_0}{h} = \frac{1941 - 1941}{10} = \frac{1}{2} = 0.5$

$$\therefore y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots$$

$$y(1946) = 20 + (0.5)4 + \frac{0.5(-0.5)}{2} + \frac{(0.5)(-0.5)(-1.5)}{6}$$

$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24}(0) + \frac{0.5(-0.5)(-1.5)(-2.5)(-3.5)}{120}(-9)$$

$$= 20 + 2 - 0.125 + 0.0625 - 0.2461$$

$$y(1946) = 21.6914$$

ii). $x = 1976$, we use backward difference formula.

Here $v = \frac{x - x_n}{h} = \frac{1976 - 1991}{10} = -1.5$

$$\therefore y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n$$

$$+ \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \frac{v(v+1)(v+2)(v+3)(v+4)}{5!}\nabla^5 y_n$$

$$= 51 - 1.5(5) + \frac{(-1.5)(-0.5)}{2}(-5) + \frac{(-1.5)(-0.5)(0.5)}{6}(-8)$$

$$+ \frac{(-1.5)(-0.5)(0.5)(1.5)}{24}(-9) + \frac{(-1.5)(-0.5)(0.5)(1.5)(2.5)}{120}(-9)$$

$$= 51 - 7.5 - 1.875 - 0.5 - 0.2109 - 0.1055$$

$$y = 40.8086$$

Q. Find the values of y at $x=21$ and $x=28$ from the following data.

| | | | | |
|---|---|---|---|---|
| $x$ : | 20 | 23 | 26 | 29 |
| $y$ : | 0.3420 | 0.3907 | 0.4384 | 0.4848 |

Soln.

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 20 | 0.3420 | | | |
| | | 0.0487 | | |
| 23 | 0.3907 | | -0.001 | |
| | | 0.0477 | | -0.0003 |
| 26 | 0.4384 | | -0.0013 | |
| | | 0.0464 | | |
| 29 | 0.4848 | | | |

Here $h=3$

i). Since $x=21$ is nearer to the beginning of the table, we use newton's forward formula.

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

Here $u = \frac{x-x_0}{h} = \frac{21-20}{3} = 0.3333$

$$\therefore y(21) = 0.3420 + (0.3333)(0.0487) + \frac{0.3333(-0.6667)(-0.001)}{2}$$

$$+ \frac{0.3333(-0.6667)(-1.6667)(-0.0003)}{6}$$

$= 0.3420 + 0.0162 + 0.0001 - 0.$

$= 0.3583$

ii). Since $x=28$ is nearer to the end value of the table, we use newton's backward interpolation formula.

Here $v = \frac{x-x_n}{h} = \frac{28-29}{3} = -0.3333$

$$\therefore y(28) = 0.4848 + (-0.3333)(0.0464) + \frac{(-0.3333)(0.6667)(-0.0013)}{2} + \frac{(-0.3333)(0.6667)(1.6667)(-0.0003)}{6}$$

$= 0.4848 - 0.01547 + 0.00014 + 0$

$y(28) = 0.4695$

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3]. From the following data, find $\theta$ at $x = 43$ and $x = 84$

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$: | 40 | 50 | 60 | 70 | 80 | 90 |
| $\theta$: | 184 | 204 | 226 | 250 | 276 | 304 |

Soln.

| $x$ | $\theta$ | $\Delta\theta$ | $\Delta^2\theta$ | $\Delta^3\theta$ |
|---|---|---|---|---|
| 40 | 184 | | | |
| | | 20 | | |
| 50 | 204 | | 2 | |
| 60 | 226 | 22 | | 0 |
| 70 | 250 | 24 | 2 | |
| 80 | 276 | 26 | 2 | 0 |
| 90 | 304 | 28 | 2 | 0 |

Here $h = 10$.

To find $x = 43$, we use Newton's forward difference formula.

$$\theta(x) = \theta_0 + \frac{u}{1!}\Delta\theta_0 + \frac{u(u-1)}{2!}\Delta^2\theta_0 + \dots$$

Here $u = \frac{x - x_0}{h} = \frac{43-40}{10} = 0.3$

$$\therefore \theta(43) = 184 + (0.3)(20) + \frac{(0.3)(-0.7)(2)}{2}$$

$$= 184 + 6 - 0.21$$

$$= 189.79$$

To find $x = 84$, we use Newton's Backward difference formula.

$$\theta(x) = \theta_n + V\nabla\theta_n + \frac{V(V+1)}{2!}\nabla^2\theta_n + \dots$$

Here $V = \frac{x - x_n}{h} = \frac{84-90}{10} = -0.6$

$$\therefore \theta(84) = 304 + (-0.6)(28) + \frac{(-0.6)(0.4)}{2}(2) + 0$$

$$= 304 - 16.8 - 0.24$$

$$\theta(84) = 286.96$$

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HW

4. From the following table, find the value of $\tan 45^\circ 15'$

$h = 1^\circ$ — 1 degree = 60 min

1.008763

| | | | | | |
|---|---|---|---|---|---|
| $x^\circ$ : | 45 | 46 | 47 | 48 | 49 |
| $\tan x^\circ$ : | 1.0000 | 1.03553 | 1.07237 | 1.11061 | 1.15037 |