



DEPARTMENT OF MECHANICAL ENGINEERING

QUANTITATIVE ABILITY III

Train problems- Boats and streams

Question 1: A train passes two bridges of length 1000 m and 600 m in 120 seconds and 80 seconds respectively. The length of the train.

Solution: Distance covered in 120 second = 1000 + length of train(l)

Distance covered in 80 seconds = 600 + l

So, distance covered in 40 seconds = (1000 + l) – (600 + l)
= 400 m

Speed = 400/40 = 10 m/s

Distance covered in 80 second = 80 x 10 = 800 m

So, 600 + l = 800

Length of the train (l) = **200 m**

Question 2: A train 500 m long is running at a speed of 72 km/hr. If it passes through a tunnel in 50 seconds, then the length of the tunnel is :

Solution: First convert speed in m/s

So, speed = 72 x (5/18)

= 20 m/s

Train covers the distance in 50 seconds = length of train + length of the tunnel(l)

500 + l = 20 x 50

500 + l = 1000

l = **500 m**

Question 3: A train reaches from A to B in 5 hours travelling at a speed of 60 km/hr. If its speed is increased by 15 km/hr, then the time of journey is reduced by

Solution: Total distance = speed x time

= 60 x 5 = 300 km

If speed increased then new speed = 60 + 15 = 75 km/hr

New time = Total distance/speed

= 300/75 = 4 hour

Time reduced by 5 – 4 = **1 hour**

Question 4: Delhi and Mumbai apart from each other 760 km. A train starts from Delhi at 9 am and travels towards Mumbai at speed 60 km/hr. Another train starts from Mumbai at 10 am and travels towards Delhi at speed 80 km/hr. At what time both will meet?

Solution: Total distance between D and M = 760 km.

A travels 1 hour before B so it travels = 60 x 1 = 60 km

Now the remaining distance D and M = 760 – 60 = 700 km

Relative speed = 60 + 80 = 140 km/hr

Time = 700 / 140

= 5 hour.

So, the time when they meet = 10 am + 5 hour = **3 pm**

Question 5: Two trains 180 m and 120 m long respectively pass each other in 54 seconds when they run in the same direction and in 18 seconds when run in opposite directions. Find the speed of two trains.

Solution: Let the speed of 1st train is S_1 and speed of 2nd train is S_2

Time = total distance/ relative speed

1) In same direction

$$54 = (180 + 120) / (S_1 - S_2) * 5/18$$

$$(S_1 - S_2)54 = (300 * 18)/5$$

$$(S_1 - S_2) = 20$$

2) In opposite direction

$$9 = (180 + 120) / (S_1 + S_2) * 5/18$$

$$(S_1 + S_2)18 = (300 * 18)/5$$

$$(S_1 + S_2) = 60$$

from 1 and 2

$$S_1 = \mathbf{40 \text{ km/hr}}$$

$$S_2 = \mathbf{20 \text{ km/hr}}$$

Question 6: Two trains start from station A and B and travels towards each other at speed of 48km/hr and 72km/hr respectively. At the time of their meeting, the second train has traveled 144 km more than the first. The distance between A and B is:

Solution: The second train has traveled 144 km more than the first train because the speed of second train is 24 km/hr more than first.

Time taken by second train to cover 144 km with surplus 24km/hr = $144/24 = 6$ hours.

then, time taken by both train before meeting is 6 hours.

So, their relative speed = $48 + 72 = 120$

Total distance travel by both = $120 \times 6 = 720$ km

Distance between A and B = **720 km**

Question 7: If the speed of the boat in still water is 5 km/hr and the speed of the current is 10 km/hr, then find the time taken by the boat to travel 125 km with the current.

Solution: Relative speed = $5 + 10$

$$= 15 \text{ km/hr}$$

Time = Distance/speed

$$= 125/15$$

$$= \mathbf{8.34 \text{ hour}}$$

Question 8: On a river, C is the mid-point between two points A and B on the same bank of the river. A boat can go from A to C and back in 14 hours and from A to B in 20 hours 20 min. How long it would take to go from B to A?

Solution: Time required to travel from A to B = 20 hour 20 min

Time required to travel from A to C = $1/2$ (20 h 20 m)

$$= 10 \text{ h } 10 \text{ m}$$

Given total time from A to C and C to A = 14 h

$$10 \text{ h } 10 \text{ m} + \text{C to A} = 14 \text{ h}$$

$$\text{C to A} = 3 \text{ h } 50 \text{ m}$$

Time taken from B to A is twice of C to A

$$\text{then, time taken from B to A} = 2 * (3 \text{ h } 50 \text{ m}) = \mathbf{7 \text{ h } 40 \text{ m}}$$

Question 9: The ratio of speed of a motor-boat to that of the current of water is 17 : 5. The boat goes along with the current in 4 hours. It will come back in

Solution: Since the ratio 17 : 5 is given.

Let the speed of boat in still water = 17 km/hr and speed of stream = 5 km/hr

Downstream speed = $17 + 5 = 22$ km/hr

Upstream speed = $17 - 5 = 12$ km/hr

Distance = Downstream speed x downstream time

= $22 \times 4 = 88$ km

Upstream time = Distance/upstream speed

= $88/12$

Come back time = **7 hour 20 minute**

Question 10: Speed of motorboat in still water is 35 kmph. If the motorboat travels 100 km along the stream in 2 hour 30 min, then the time taken by it to cover the same distance against the stream is

Solution: The speed of the motorboat in still water is 35 km/hr.

let the speed of the stream = x km/hr

Downstream speed = Distance/time

= $100 / 2.5$

= 40 km/hr

Speed of stream = $35 + x = 40$

$x = 5$ km/hr

Upstream speed = $35 - 5 = 30$ km/hr

Time taken in upstream = $100/30 = \mathbf{3 \text{ hour } 20 \text{ min}}$

Pipes and cisterns

Question 1 : Two pipes A and B can fill a tank separately in 12 and 16 hours respectively. If both of them are opened together when the tank is initially empty, how much time will it take to completely fill the tank?

Solution : Part of tank filled by pipe A in one hour working alone = $1 / 12$

Part of tank filled by pipe B in one hour working alone = $1 / 16$

=> Part of tank filled by pipe A and pipe B in one hour working together = $(1 / 12) + (1 / 16) = 7 / 48$

Therefore, time taken to completely fill the tank if both A and B work together = $48 / 7$ hours

Another Method

Let the capacity of tank be LCM (12, 16) = 48 units

=> Efficiency of pipe A = $48 / 12 = 4$ units / hour

=> Efficiency of pipe B = $48 / 16 = 3$ units / hour

=> Combined efficiency of pipes A and B = 7 units / hour

Therefore, time taken to completely fill the tank = $48 / 7$ hours

Question 2 : Three pipes A, B and C are connected to a tank. Out of the three, A and B are the inlet pipes and C is the outlet pipe. If opened separately, A fills the tank in 10 hours, B fills the tank in 12 hours and C empties the tank in 30 hours. If all three are opened simultaneously, how much time does it take to fill / empty the tank ?

Solution : Part of tank filled by pipe A in one hour working alone = $1 / 10$

Part of tank filled by pipe B in one hour working alone = $1 / 12$

Part of tank emptied by pipe C in one hour working alone = $1 / 30$

=> Part of tank filled by pipes A,B and C in one hour working together = $(1 / 10) + (1 / 12) - (1 / 30) = 3 / 20$

Therefore, time taken to completely fill the tank if both A and B work together = $20 / 3$ hours = 6 hours 40 minutes

Another Method

Let the capacity of tank be LCM (10, 12, 30) = 60 units

=> Efficiency of pipe A = $60 / 10 = 6$ units / hour

=> Efficiency of pipe B = $60 / 12 = 5$ units / hour

=> Efficiency of pipe C = $- 60 / 30 = - 2$ units / hour (Here, '-' represents outlet pipe)

=> Combined efficiency of pipes A, B and C = $6 + 5 - 2 = 9$ units / hour

Therefore, time taken to completely fill the tank = $60 / 9 = 6$ hours 40 minutes

Question 3 : Three pipes A, B and C are connected to a tank. Out of the three, A is the inlet pipe and B and C are the outlet pipes. If opened separately, A fills the tank in 10 hours, B empties the tank in 12 hours and C empties the tank in 30 hours. If all three are opened simultaneously, how much time does it take to fill / empty the tank ?

Solution : Part of tank filled by pipe A in one hour working alone = $1 / 10$

Part of tank emptied by pipe B in one hour working alone = $1 / 12$

Part of tank emptied by pipe C in one hour working alone = $1 / 30$

=> Part of tank filled by pipes A, B and C in one hour working together = $(1 / 10) - (1 / 12) - (1 / 30) = -1 / 60$

Therefore, time taken to completely empty the tank if all pipes are opened simultaneously = $1 / 60$ hours = 60 hours

Another Method

Let the capacity of tank be LCM (10, 12, 30) = 60 units

=> Efficiency of pipe A = $60 / 10 = 6$ units / hour

=> Efficiency of pipe B = $- 60 / 12 = - 5$ units / hour (Here, '-' represents outlet pipe)

=> Efficiency of pipe C = $- 60 / 30 = - 2$ units / hour (Here, '-' represents outlet pipe)

=> Combined efficiency of pipes A, B and C = $6 - 5 - 2 = - 1$ units / hour (Here, '-' represents outlet pipe)

Therefore, time taken to completely empty the tank = $60 / (1) = 60$ hours

Question 4 : A cistern has two pipes. Both working together can fill the cistern in 12 minutes. First pipe is 10 minutes faster than the second pipe. How much time would it take to fill the cistern if only second pipe is used ?

Solution : Let the time taken by first pipe working alone be 't' minutes.

=> Time taken by second pipe working alone = $t + 10$ minutes.

Part of tank filled by pipe A in one hour working alone = $1 / t$

Part of tank filled by pipe B in one hour working alone = $1 / (t + 10)$

=> Part of tank filled by pipe A and B in one hour working together = $(1 / t) + (1 / t+10) = (2t + 10) / [t x (t + 10)]$

But we are given that it takes 12 minutes to completely fill the cistern if both pipes are working together.

=> $(2t + 10) / [t x (t + 10)] = 1 / 12$

=> $t x (t + 10) / (2t + 10) = 12$

=> $t^2 + 10t = 24t + 120$

=> $t^2 - 14t - 120 = 0$

=> $(t - 20) (t + 6) = 0$

$\Rightarrow t = 20$ minutes (Time cannot be negative)

Therefore, time taken by second pipe working alone = $20 + 10 = 30$ minutes

Another Method

Let the time taken by first pipe working alone be 't' minutes.

\Rightarrow Time taken by second pipe working alone = $t + 10$ minutes.

Let the capacity of cistern be $t \times (t + 10)$ units.

\Rightarrow Efficiency of first pipe = $t \times (t + 10) / t = (t + 10)$ units / minute

\Rightarrow Efficiency of second pipe = $t \times (t + 10) / (t + 10) = t$ units / minute

\Rightarrow Combined efficiency of pipes = $(2t + 10)$ units / minute

\Rightarrow Time taken to fill the cistern completely = $t \times (t + 10) / (2t + 10)$

But we are given that it takes 12 minutes to completely fill the cistern if both pipes are working together.

$\Rightarrow t \times (t + 10) / (2t + 10) = 12$

$\Rightarrow t^2 + 10t = 24t + 120$

$\Rightarrow t^2 - 14t - 120 = 0$

$\Rightarrow (t - 20)(t + 6) = 0$

$\Rightarrow t = 20$ minutes (Time cannot be negative)

Therefore, time taken by second pipe working alone = $20 + 10 = 30$ minutes

Question 5 : Three pipes A, B and C are connected to a tank. Out of the three, A and B are the inlet pipes and C is the outlet pipe. If opened separately, A fills the tank in 10 hours and B fills the tank in 30 hours. If all three are opened simultaneously, it takes 30 minutes extra than if only A and B are opened. How much time does it take to empty the tank if only C is opened?

Solution : Let the capacity of tank be LCM (10, 30) = 30 units

\Rightarrow Efficiency of pipe A = $30 / 10 = 3$ units / hour

\Rightarrow Efficiency of pipe B = $30 / 30 = 1$ units / hour

\Rightarrow Combined efficiency of pipes A and B = 4 units/hour

Therefore, time taken to completely fill the tank if only A and B are opened = $30 / 4 = 7$ hours 30 minutes

\Rightarrow Time taken to completely fill the tank if all pipes are opened = 7 hours 30 minutes + 30 minutes = 8 hours

\Rightarrow Combined efficiency of all pipes = $30 / 8 = 3.75$ units / hour

Now, efficiency of pipe C = Combined efficiency of all three pipes – Combined efficiency of pipes A and B

Therefore, efficiency of pipe C = $4 - 3.75 = 0.25$ units / hour

Thus, time taken to empty the tank if only C is opened = $30 / 0.25 = 120$ hours

Question 6 : Time required by two pipes A and B working separately to fill a tank is 36 seconds and 45 seconds respectively. Another pipe C can empty the tank in 30 seconds. Initially, A and B are opened and after 7 seconds, C is also opened. In how much more time the tank would be completely filled ?

Solution : Let the capacity of the tank be LCM (36, 45, 30) = 180 units

\Rightarrow Efficiency of pipe A = $180 / 36 = 5$ units / second

\Rightarrow Efficiency of pipe B = $180 / 45 = 4$ units / second

\Rightarrow Efficiency of pipe C = $-180 / 30 = -6$ units / second

Now, for the first 7 seconds, A and B were open.

\Rightarrow Combined efficiency of A and B = $5 + 4 = 9$ units / second

\Rightarrow Part of the tank filled in 7 seconds = $7 \times 9 = 63$ units

=> Part of tank empty = $180 - 63 = 117$ units

Now, all pipes are opened.

=> Combined efficiency of all pipes = $5 + 4 - 6 = 3$ units / second

Therefore, more time required = $117 / 3 = 39$ seconds

Question 7 : Two pipes A and B can fill a tank in 20 hours and 30 hours respectively. If both the pipes are opened simultaneously, find after how much time should pipe B be closed so that the tank is full in 18 hours?

Solution : Let the capacity of the tank be $\text{LCM}(20, 30) = 60$ units

=> Efficiency of pipe A = $60 / 20 = 3$ units / hour

=> Efficiency of pipe B = $60 / 30 = 2$ units / hour

=> Combined efficiency of pipes A and B = 5 units / hour

Let both A and B be opened for 'n' hours and then, B be closed and only A be opened for the remaining '18 - n' hours.

=> $5n + 2 \times (18 - n) = 60$

=> $3n + 36 = 60$

=> $3n = 24$

=> $n = 8$

Therefore, B should be closed after 8 hours.