TRIPLE INTEGRATION

① Evaluate $\int_0^a \int_0^b \int_0^c xyz \, dz \, dy \, dx$.

$$= \int_0^a \int_0^b xy \left[\frac{z^2}{2}\right]_0^c dy \, dx$$

$$= \int_0^a \int_0^b xy \frac{c^2}{2} \, dy \, dx$$

$$= \frac{c^2}{2} \int_0^a x \left[\frac{y^2}{2}\right]_0^b dx \quad = \frac{c^2}{2} \int_0^a x \frac{b^2}{2} \, dx$$

$$= \frac{b^2 c^2}{4} \int_0^a x \, dx \quad = \frac{b^2 c^2}{4} \left[\frac{x^2}{2}\right]_0^a$$

$$= \frac{a^2 b^2 c^2}{8}.$$

$$\int_0^1 \int_0^2 \int_0^3 xyz \ dz\, dy\, dx$$

$$= \int_0^1 \int_0^2 xy \left[\frac{z^2}{2}\right]_0^3 dy\, dx$$

$$= \int_0^1 \int_0^2 xy\left(\frac{9}{2}\right) dy\, dx$$

$$= \frac{9}{2} \int_0^1 \int_0^2 xy \ dy\, dx$$

$$= \frac{9}{2} \int_0^1 x\left[\frac{y^2}{2}\right]_0^2 dx$$

$$= \frac{9}{2} \int_0^1 x\left(\frac{4}{2}\right) dx$$

$$= 2 \times \frac{9}{2} \int_0^1 x \ dx$$

$$= 9 \left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{9}{2}$$

$$\int_0^1 \int_0^2 \int_0^3 e^{x+y+z} \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^2 \int_0^3 e^x . e^y . e^z \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^2 e^x e^y \left[e^z \right]_0^3 \, dy \, dx$$

$$= (e^3 - e^0) \int_0^1 \int_0^2 e^x e^y \, dy \, dx$$

$$= (e^3 - 1) \int_0^1 e^x (e^y)_0^2 \, dx$$

$$= (e^3 - 1)(e^2 - e^0) \int_0^1 e^x \, dx$$

$$= (e^3 - 1)(e^2 - 1)(e^1 - 1).$$