



DEPARTMENT OF MATHEMATICS

UNIT – I MULTIPLE INTEGRALS

TRIPLE INTEGRATION

① Evaluate $\int_0^a \int_0^b \int_0^c xyz \, dz \, dy \, dx$.

$$= \int_0^a \int_0^b xy \left[\frac{z^2}{2} \right]_0^c dy \, dx$$

$$= \int_0^a \int_0^b xy \frac{c^2}{2} dy \, dx$$

$$= \frac{c^2}{2} \int_0^a x \left[\frac{y^2}{2} \right]_0^b dx = \frac{c^2}{2} \int_0^a x \frac{b^2}{2} dx$$

$$= \frac{b^2 c^2}{4} \int_0^a x \, dx = \frac{b^2 c^2}{4} \left[\frac{x^2}{2} \right]_0^a$$

$$= \frac{a^2 b^2 c^2}{8}$$



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$$\begin{aligned} & \int_0^1 \int_0^2 \int_0^3 xyz \, dz dy dx \\ &= \int_0^1 \int_0^2 xy \left[\frac{z^2}{2} \right]_0^3 dy dx \\ &= \int_0^1 \int_0^2 xy \left(\frac{9}{2} \right) dy dx \\ &= \frac{9}{2} \int_0^1 \int_0^2 xy \, dy dx \\ &= \frac{9}{2} \int_0^1 \left[\frac{xy^2}{2} \right]_0^2 dx \\ &= \frac{9}{2} \int_0^1 x \left(\frac{4}{2} \right) dx \\ &= \frac{9}{2} \int_0^1 x \, dx \\ &= 9 \left[\frac{x^2}{2} \right]_0^1 \\ &= \frac{9}{2} \end{aligned}$$



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$$\begin{aligned} & \int_0^1 \int_0^2 \int_0^3 e^{x+y+z} dz dy dx \\ &= \int_0^1 \int_0^2 \int_0^3 e^x \cdot e^y \cdot e^z dz dy dx \\ &= \int_0^1 \int_0^2 e^x e^y [e^z]_0^3 dy dx \\ &= (e^3 - e^0) \int_0^1 \int_0^2 e^x e^y dy dx \\ &= (e^3 - 1) \int_0^1 e^x (e^y)_0^2 dx \\ &= (e^3 - 1) (e^2 - e^0) \int_0^1 e^x dx \\ &= (e^3 - 1) (e^2 - 1) (e^1 - 1). \end{aligned}$$