

(An Autonomous Institution) DEPARTMENT OF MATHEMATICS



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NUMERICAL DIFFERENTIATION & INTEGRATION

NUMERICAL DIFFERENTIATION :

It is the process of computing the value of the desivative dy for some particular value of x, from the given data (x;, y;). If the values of x are equally spaced, we can use Newton's interpolation formula for eavual intervals. If the values of x are unequally spaced, we can use Lagrange's interpolation formula (or) Newton's divided difference Interpolation formula. Differentiation Using interpolation formulae: Newton's forward difference formula to compute the derivatives : Let us consider Newton's forward difference formula, $y = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^$ $\frac{u(u-1)(u-2)(u-3)}{4!} \Delta^{4} y_{0} + \cdots$ where $u = \frac{\chi - \chi_0}{h}$ $i \cdot e \cdot , y = y_0 + u \wedge y_0 + (u^2 - u) \wedge^2 y_0 + (u^3 - 3u^2 + 2u) \wedge^3 y_0$ $+ (u^4 - 6u^3 + 11u^2 - 6u) \Delta^{+} y_0 + \cdots$ Now, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$





$$\frac{dy}{dx} = \frac{1}{h} \frac{dy}{du}$$
i.e. $\frac{dy}{dx} = \frac{1}{h} \begin{cases} \Delta y_{0} + \frac{(3u-1)}{2} \Delta^{2} y_{0} + \frac{(3u^{2}-bu+3)}{b} \Delta^{3} y_{0} + \frac{(4u^{3}-18u^{2}+3u-b)}{b} \Delta^{3} y_{0} + \frac{(4u^{3}-18u^{2}+3u-b)}{b} \Delta^{4} y_{0} + \cdots \frac{1}{y}$

$$\frac{d^{2}y}{dx^{2}} = \frac{1}{h^{2}} \begin{cases} \Delta^{2} y_{0} + (u-1)\Delta^{3} y_{0} + \frac{(bu^{2}-18u+11)}{12} \Delta^{4} y_{0} + \cdots \frac{1}{y} \rightarrow \odot$$

$$\frac{d^{3}y}{dx^{3}} = \frac{1}{h^{3}} \begin{cases} \Delta^{3} y_{0} + \frac{18u-18}{12} \Delta^{4} y_{0} + \cdots \frac{1}{y} \rightarrow \odot$$

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$$\frac{d^{3}y}{dx^{3}} = \frac{1}{h^{3}} \begin{cases} \Delta^{3} y_{0} + \frac{18u-18}{12} \Delta^{4} y_{0} - \frac{1}{2} \Delta^{2} y_{0} + \frac{\Delta^{3} y_{0}}{3} - \frac{\Delta^{4} y_{0} + \cdots \frac{1}{y}} \rightarrow \odot$$

$$\frac{d^{3}y}{dx^{3}} = \frac{1}{h^{3}} \begin{pmatrix} \frac{d^{2}y}{dx^{3}} \end{pmatrix}_{u=0} = \frac{1}{h} \begin{cases} \Delta^{3} y_{0} - \frac{1}{2} \Delta^{2} y_{0} + \frac{\Delta^{3} y_{0}}{3} - \frac{\Delta^{4} y_{0} - \cdots \frac{1}{y}} \rightarrow \odot$$

$$\frac{d^{4}y_{0}}{(dx^{3})} = \frac{1}{x^{2}x_{0}} \begin{pmatrix} \frac{\Delta^{3}y}{dx^{2}} \end{pmatrix}_{u=0} = \frac{1}{h^{3}} \begin{cases} \Delta^{3} y_{0} - \frac{3}{2} \Delta^{3} y_{0} + \frac{1}{12} \Delta^{3} y_{0} - \frac{1}{y}} \rightarrow \odot$$

$$\frac{d^{3}y}{(dx^{3})} = \frac{1}{h^{3}} \begin{pmatrix} \Delta^{3} y_{0} - \frac{3}{2} \Delta^{4} y_{0} + \cdots \frac{1}{y}} \rightarrow \odot$$

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Newton's Backward Difference formula to compute the
derivatives:

$$\frac{dy}{dx} = \frac{1}{h} \left\{ \nabla y_n + \frac{2v+1}{d} \nabla^2 y_n + \frac{(3v^2+bv+a)}{b} \nabla^3 y_n + \frac{4v^3+(8v^2+2av+b)}{2v} \nabla^3 y_n + \frac{4v^3+(8v^2+2av+b)}{2v} \nabla^4 y_n + \cdots \right\}$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2} \left\{ \nabla^2 y_n + (v+1) \nabla^3 y_n + \frac{bv^2+18v+11}{12} \nabla^4 y_n + \cdots \right\}$$

$$\frac{d^3 y}{dx^2} = -\frac{1}{h^3} \left\{ \nabla^3 y_n + \frac{12v+18}{12} \nabla^4 y_n + \cdots \right\}$$
The particular, at $x = x_n$, $v = 0$. Then

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h} \left\{ \nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{2} + \frac{\nabla^4 y_n}{4} + \cdots \right\}$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2} \left\{ \nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \cdots \right\}$$

$$\left(\frac{d^3 y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3} \left\{ \nabla^3 y_n + \frac{3}{2} \nabla^4 y_n + \cdots \right\}$$



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Problems: population of a certain town is given below. (1) The Find the rate of growth of the population in 1931. 1941, 1961 and 1971. Year: 1931 1941 1951 1961 1971 : 40.62 60.80 79.95 103.56 132.65 Population in thousands y Solution : 1º 4 $\Delta^2 y \quad \Delta^3 y$ Δ4 y x 1931 40.62 20.18 -1.03 60.80 1941 5.49 19.15 4.46 79.95 1951 23.61 1.02 5.48 . 103.56 1961 29.09 132.65 1971 (i) To get f'(1931) and f'(1941) we use forward formula. $U = \frac{\chi - \chi_0}{h} = \frac{1931 - 1931}{10} = 0$ $\left(\frac{dy}{dx}\right) = \frac{1}{h} \begin{cases} \Delta y_0 - \frac{\Delta^2 y_0}{a} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots \end{cases}$ $= \frac{1}{10} \left\{ 20.18 - \frac{(-1.03)}{2} + \frac{(5.49)}{3} - \frac{(-4.47)}{4} \right\}$







(3)

$$\left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{10} \begin{cases} 20.18 + 0.515 + 1.83 + 1.1175 \end{cases}$$

= 2.3643

(i) To find
$$y'(1941)$$
:

$$U = \frac{\chi - \chi_{0}}{h} = \frac{1941 - 1931}{10} = 1$$

$$\frac{dy}{dx} = \frac{1}{h} \left\{ \Delta y_{0} + \frac{20-1}{2} \Delta^{2} y_{0} + \frac{3u^{2} - 6u + 2}{6} \Delta^{3} y_{0} + \frac{4u^{3} - 18u^{2} + 22u - 6}{6} \Delta^{4} y_{0} + \cdots + \frac{3}{24} \right\}$$

$$= \frac{1}{10} \left\{ 20.18 + \frac{1}{2} (-1.03) - \frac{1}{6} (5.49) + \frac{1}{12} (-4.97) \right\}$$

$$= \frac{1}{10} \left\{ 20.18 - 0.515 - 0.915 - 0.3735 \right\}$$

$$= \frac{1.833755}{10}$$
(iii) To find $y'(1961)$ and $y'(1971)$ we use Newtow's backward formula

$$V = \frac{\chi - \chi_{0}}{h} = \frac{1960 - 1971}{10} = -1$$

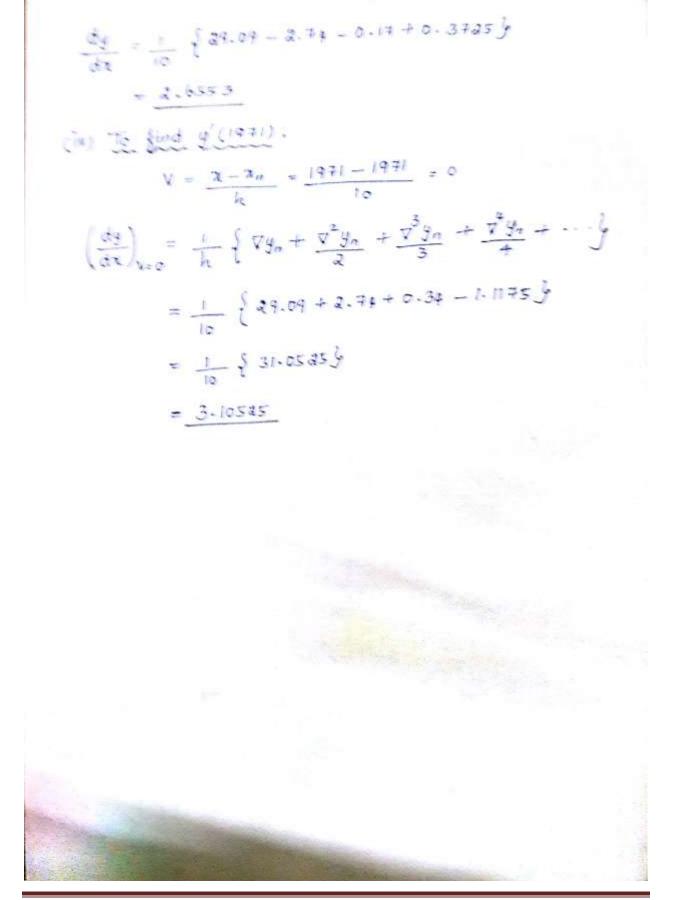
$$\frac{dy}{dx} = \frac{1}{h} \left[\nabla y_{0} + \frac{2v+1}{2} \nabla^{2} y_{0} + \frac{3v^{2} + 6v+2}{6} \nabla^{3} y_{0} + \frac{4v^{3} + 18v^{2} + 22v+6}{6} \nabla^{4} y_{0} + \cdots \right]$$

$$= \frac{1}{10} \left[29.09 - \frac{1}{2} (5.48) - \frac{1}{6} (1.02) - \frac{1}{2} (-4.97) \right]$$



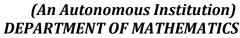
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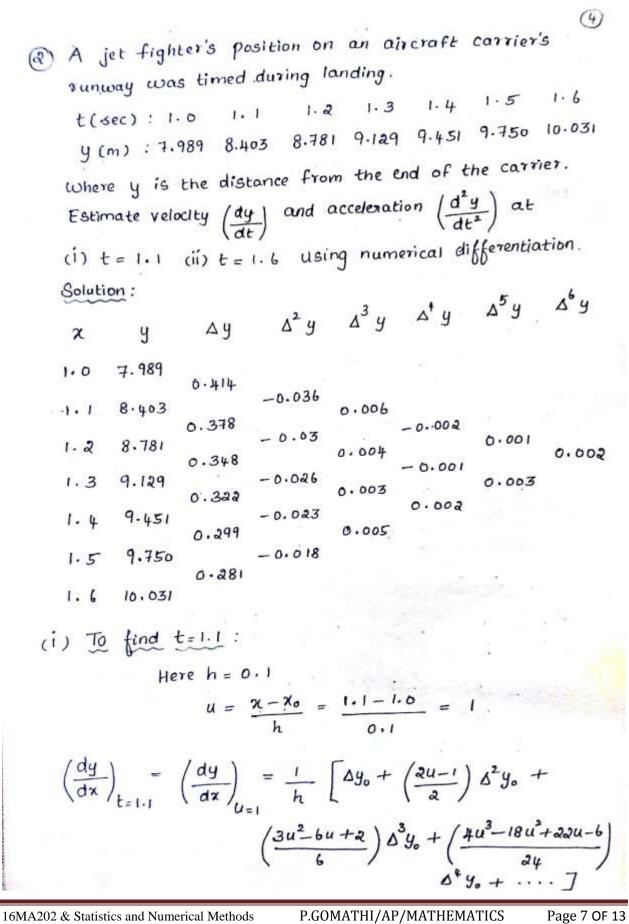


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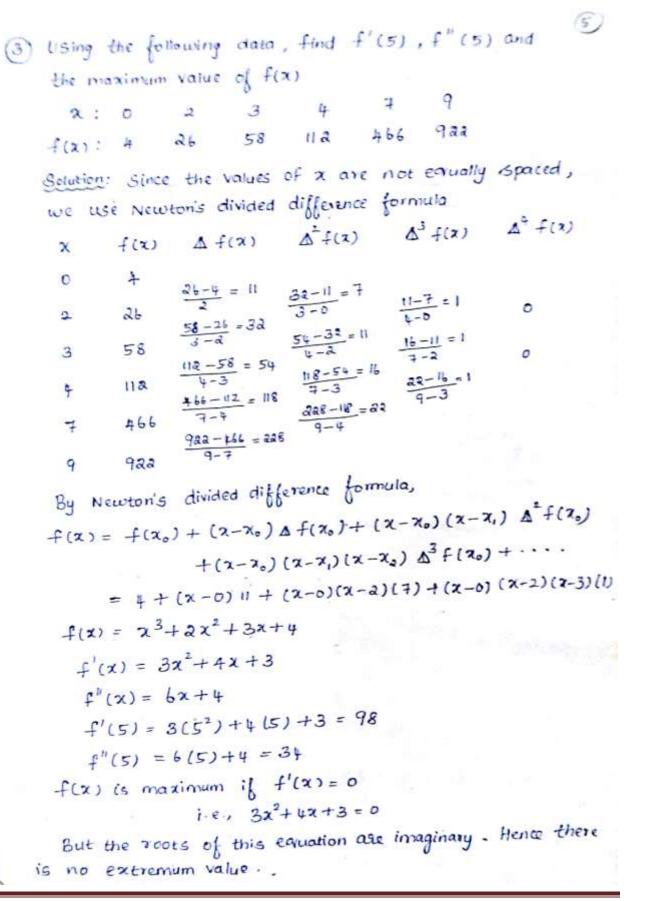


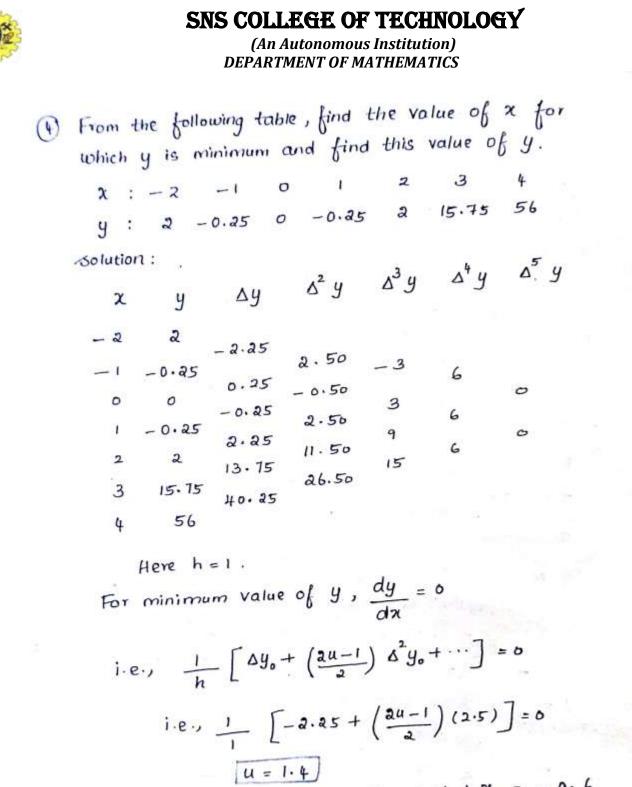












$$\begin{array}{rcl} & u = 1 \cdot 4 \\ \hline u = 1 \cdot 4 \\ \hline & u = 1 \cdot 4 \\ \hline & \chi - \chi_{0} \\ \hline & h \end{array} = 1 \cdot 4 \implies \chi = 1 \cdot 4h + \chi_{0} = -0 \cdot 6 \\ \hline & \chi - \chi_{0} \\ \hline & h \end{array}$$

$$\begin{array}{rcl} & y = 1 \cdot 4 \\ \hline & h \end{array} \implies \chi = 1 \cdot 4h + \chi_{0} = -0 \cdot 6 \\ \hline & \chi - \chi_{0} \\ \hline & h \end{array}$$

$$\begin{array}{rcl} & y = 1 \cdot 4 \\ \hline & \chi - \chi_{0} \\ \hline & h \end{array}$$

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