Test of Significance of Small Samples:

Student's t - Test.

Test for single mean:

Null hypothesis: $H_0 : \mu = \mu_0$

Test Statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \quad \text{if SD is given}$$

(or)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{if SD is not given}$$

To find s:

$$s^2 = \frac{\Sigma (x - \bar{x})^2}{n-1} \quad ; \text{ Degrees of freedom } \nu = n-1$$

Application of t_test:

* To test the significance of a single mean
* To test the significance of the difference between 2 sample mean.
* To test the significance of the coefficient of correlation.

Assumption for Student's t-test.

i). The

1. A random sample of 10 boys had the following IQ's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ's of 100? Find a reasonable range in which most of the mean IQ's value of sample 10 boys.

Soln.

Given $n = 10$, $\mu = 100$

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$= 97.2$

To find s:

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

x	70	120	110	101	88	83	95	98	107	100
$x-\bar{x}$	-27.2	22.8	12.8	3.8	-9.2	-14.2	-2.2	0.8	9.8	2.8
$(x-\bar{x})^2$	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84

$\Sigma(x-\bar{x})^2 = 1833.6$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1833.6}{10-1}$$

$s^2 = 203.73$

$\Rightarrow s = 14.27$

Scanned with CamScanner

Step 1:

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$ (Two tailed test)

Step 2:

LOS, $\alpha = 5\%$

Step 3:

Test Statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{97.2 - 100}{14.27/\sqrt{10}}$$

$$= -0.62$$

$|t| = 0.62$

Step 4:

Critical value for degrees of freedom

$\nu = n - 1$

$= 10 - 1 = 9$

ie, $t_{tab} = 2.262 \ (t_\alpha)$

Step 5:

Conclusion:

Calculated value < table value

$\therefore$ H_0 is accepted.

ie, The population mean IQ's is 100.

Confidence limit:

$$\mu = \bar{x} \pm t_\alpha \frac{s}{\sqrt{n-1}} = 97.2 \pm 2.262 \times \frac{14.27}{\sqrt{10-1}}$$

$= 97.2 \pm 10.759$

$= 107.95, 86.45$

Scanned with CamScanner

27. The weights of 10 peoples of a locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 kg. It is reasonable to believe that the avg. weights of people locality greater than 64 kg. Test at 5% LOS

Soln.

Given $n = 10 < 30$ (Small Sample)

$\mu = 64$

$$\bar{x} = \frac{70+67+62+68+61+68+70+64+64+66}{10}$$

$\bar{x} = 66$

To find S:

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
70	4	16
67	1	1
62	-4	16
68	2	4
61	-5	25
68	2	4
70	4	16
64	-2	4
64	-2	4
66	0	0

$\Sigma(x-\bar{x})^2 = 90$

$$\therefore S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{90}{10-1} = 10$$

$S = 3.16$

Step 1:

$H_0 : \mu = 64$

$H_1 : \mu > 64$ (one tailed test - Right)

Step 2:

LOS, $\alpha = 5\%$

Step 3:

Test Statistic :

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{66 - 64}{3.16/\sqrt{10}}$$

$$= 2.02$$

Step 4:

Degrees of freedom $\nu = n - 1$

$= 10 - 1$

$= 9$

$\therefore t_{tab} : t_\alpha = 1.833$ &

Step 5:

conclusion :

Calculated value > Table value

$\therefore H_0$ is rejected.

Hence the avg. weights of people locality greater than 64 kg.

3. A Sample of 26 tubelight gives a mean life of 990 hrs with a SD of 20 hrs. The company claims that the mean life of tubelight to 1000 hrs. Is the sample upto the specification.

Soln.

Given $n = 26$, $\bar{x} = 990$, $s = 20$, $\mu = 1000$

Step 1:

$H_0 : \mu = 1000$

$H_1 : \mu < 1000$ (one tailed test – left)

Step 2:

LOS, $\alpha = 5\%$

Step 3:

$$t = \frac{\bar{x} - \mu}{\frac{S.D}{\sqrt{n-1}}}$$

$$= \frac{990 - 1000}{\frac{20}{\sqrt{26-1}}}$$

$t = -2.5$

Step 4:

Degrees of freedom $\nu = n - 1$

$= 26 - 1 = 25$

Critical region: $t_\alpha = 1.708$

Step 5:

Conclusion :

Calculated Value > Table value

$\therefore$ Ho is rejected.

Hence the sample is upto the specification.

Test for difference of mean :

Null hypothesis : $H_0 : \mu_1 = \mu_2$

Test Statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$ (or) $s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

Degree of freedom : $\nu = n_1 + n_2 - 2$

---

1. In a test examination given to two groups of Students, the marks obtained were as follows :

group I : 18 20 36 50 49 36 34 49 41

group II : 29 28 26 35 30 44 46

Examine whether the significance of difference between the average marks secured by the Students of the above two groups.

Soln.

Given : Group I : $n_1 = 9$

Group II : $n_2 = 7$

Now $\bar{x}_1 = \frac{18+20+36+50+49+36+34+49+41}{9}$

$= 37$

$\bar{x}_2 = \frac{29+28+26+35+30+44+46}{7}$

$= 34$

Scanned with CamScanner

x_1	$x_1-\bar{x}_1)$	$(x_1-\bar{x}_1)^2$	x_2	$x_2-\bar{x}_2$	$(x_2-\bar{x}_2)^2$
18	−19	361	29	−5	25
20	−17	289	28	−6	36
36	−1	1	26	−8	64
50	13	169	35	1	1
49	12	144	30	−4	16
36	−1	1	44	10	100
34	−3	9	46	12	144
49	12	144			
41	4	16			

$\Sigma(x_1-\bar{x}_1)^2 = 1134$ $\qquad$ $\Sigma(x_2-\bar{x}_2)^2 = 386$

Now

$$S^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$$

$$= \frac{1134+386}{9+7-2}$$

$$S^2 = 108.57$$

$$\Rightarrow S = 10.42$$

Step 1:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (Two tailed test)

Step 2:

LOS, $\alpha = 5\%$

Step 3:

Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$= \frac{37-34}{10.42\sqrt{\frac{1}{9}+\frac{1}{7}}}$$

$$t = 0.5713$$

Step 4 :

Critical value: Degrees of freedom

$$\nu = n_1 + n_2 - 2$$
$$= 9 + 7 - 2$$
$$\nu = 14$$

$\therefore t_\alpha = 2.145$

Step 5 :

Conclusion :

Calculated Value $<$ Table Value

$\therefore$ H_0 is Accepted.

$\therefore$ There is no significant difference in the avg. marks of the two groups of students.

2]. A samples of two types of Electric bulbs were tested for length of life and the following data were obtained.

Samples	Size	Mean	S.D
I	8	1134	35
II	7	1024	40.

Test at 5%.

Soln.

Given: Sample I : $n_1 = 8$, $\bar{x}_1 = 1134$, $S_1 = 35$

Sample II : $n_2 = 7$, $\bar{x}_2 = 1024$, $S_2 = 40$

Step 1:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$

Step 2:

LOS, $\alpha = 5\%$

Step 3:

Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{68 - 67.8}{15.257\sqrt{\frac{1}{6} + \frac{1}{10}}}$$

$t = 0.099$

Step 4:

Critical value, Degrees of freedom $= n_1 + n_2 - 2$

$= 6 + 10 - 2$

$t_\alpha = 1.761$ (Right Tailed test) $\quad \nu = 14$

Step 5:

Conclusion:

$|t| = 0.099 < 1.761 = t_\alpha|$

$\therefore$ H_0 is accepted.

$\therefore$ The sailors are not on the average taller than the soldiers.

F test for equality of variance (or) Variance Ratio test :

Null Hypothesis :

$$H_0 : \sigma_1^2 = \sigma_2^2$$

Test Statistics : $F = \frac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$

where $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ (or) $S_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1}$

and $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$ (or) $S_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1}$

Degree of freedom: (ν_1, ν_2)

where $\nu_1 = (n_1 - 1)$, $\nu_2 = (n_2 - 1)$

Note :

1) F Greater than one always

2). Suppose $S_2^2 > S_1^2$, then $F = \frac{S_2^2}{S_1^2}$

with degrees of freedom

$\nu_1 = n_2 - 1$, $\nu_2 = n_1 - 1$

Applications :

F test is used to test if the two samples have come from the same population.

1]. Two random sample of 11 and 9 items show that the sample standard deviations of their weights as 0.8 & 0.25 resly. Assuming that the weight distributions are normal, test the hypo that the true variances are equal, against the alternative hypo. that they are not.

Soln.

Given $n_1 = 11$, $S_1 = 0.8$

$n_2 = 9$, $S_2 = 0.5$

$$s_1^2 = \frac{n_1 S_1^2}{n_1 - 1} = \frac{11(0.8)^2}{11-1} = 0.704$$

$$s_2^2 = \frac{n_2 S_2^2}{n_2 - 1} = \frac{9(0.5)^2}{9-1} = 0.2812$$

$s_1^2 > s_2^2$

Step 1:

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2:

LOS at $\alpha = 5\%$

Step 3:

Test Statistic, $F = \frac{s_1^2}{s_2^2} = \frac{0.704}{0.2812} = 2.5$

Step 4:

Degrees of freedom

$(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$

$= (10, 8)$

$\therefore F_\alpha = 3.35$

Step 5:

Conclusion : $F = 2.5 < 3.32 = F_\alpha$

$\therefore H_0$ is accepted at 5%.

2] Two random samples give the following results.

Sample	Size	Sample mean	Sum of squares of deviation from the means
1	12	14	108
2	10	15	90

Test whether the samples came from the same population.

Soln.

Given $n_1 = 12$, $\bar{x}_1 = 14$, $\Sigma(x_1 - \bar{x}_1)^2 = 108$

$n_2 = 10$, $\bar{x}_2 = 15$, $\Sigma(x_2 - \bar{x}_2)^2 = 90$

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{108}{12-1} = 9.818$$

$$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{90}{10-1} = 10$$

$\therefore S_1^2 < S_2^2$

Step 1:

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2:

LOS, $\alpha = 5\%$

Step 3:

Test Statistics: $F = \frac{S_2^2}{S_1^2} = \frac{10}{9.818} = 1.018$

Step 4:

Degrees of freedom $= (\nu_1, \nu_2)$

$= (n_2 - 1, n_1 - 1)$

$= (9, 11)$

$\therefore F_\alpha = 2.9$

Step 5:

Conclusion:

$F = 1.018 < 2.90 = F_\alpha$

$\therefore$ Ho is accepted at 5% LOS.

ii). t test

Step 1:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Step 2:

LOS at $\alpha = 5\%$

Step 3:

Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Here $n_1 = 12$, $n_2 = 10$, $\bar{x}_1 = 14$, $\bar{x}_2 = 15$

Now

$$S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$= \frac{108 + 90}{12 + 10 - 2} = 9.9$$

$$S = 3.14$$

$$\therefore \quad t = \frac{14 - 15}{3.14\sqrt{\frac{1}{12} + \frac{1}{10}}}$$

$$= -0.744$$

Step 4:

Degrees of freedom, $\nu = n_1 + n_2 - 2$

$= 12 + 10 - 2$

$= 20$

$\therefore \quad t_\alpha = 2.086$

Step 5:

Conclusion:

$|t| = 0.744 < 2.086 = |t_\alpha|$

$\therefore$ H_0 is accepted at 5% LOS.

3]. Test whether the population variances are identical.

Sample I :	10	11	16	12	10	11	12	16
Sample II :	7	9	3	7	9	3	15	

at 1% LOS.

Soln:

Given $n_1 = 8$, $n_2 = 7$

x_1	$(x_1 - \bar{x}_1)^2$	x_2	$(x_2 - \bar{x}_2)^2$
10	5.0625	7	0.3265
11	1.5625	9	2.0409
16	14.0625	3	20.6977
12	0.0625	7	0.3265
10	5.0625	9	2.0409
11	1.5625	3	20.6977
12	0.0625	15	55.1841
16	14.0625		
98	41.5	53	101.7143

$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{98}{8}$ $\quad \Sigma(x_1 - \bar{x}_1)^2 = 41.5$

$\bar{x}_2 = \frac{\Sigma x_2}{n} = \frac{53}{7}$ $\quad \Sigma(x_2 - \bar{x}_2)^2 = 101.71$

$= 7.57$

$\therefore \; S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{41.5}{7} = 5.9286$

$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{101.7143}{6} = 16.9524$

$\therefore \; S_1^2 < S_2^2$

Step 1:

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2:

LOS, $\alpha = 1\%$

Step 3:

Test Statistic: $F = \frac{S_2^2}{S_1^2} = \frac{16.9524}{5.9286}$

$= 2.86$

Step 4: Degrees of freedom : (ν_1, ν_2)

$= (n_2 - 1, n_1 - 1)$

$= (6, 7)$

$\therefore F_\alpha = 7.19$

Step 5:

Conclusion :

$|F| = 2.86 < 7.19 = |F_\alpha|$

$\therefore H_0$ is accepted at 1% LOS.

Chi-Square Test :

$$\chi^2 = \sum \frac{[O_i - E_i]^2}{E_i}$$

where $O_i \rightarrow$ Observed Frequency

$E_i \rightarrow$ Expected Frequency & Experimental frequency

$$= \frac{\sum O_i}{n}$$

Degrees of freedom, $\nu = n - 1$

Properties:

i). The mean of χ^2 distribution is equal to the no of degrees of freedom.

ii). The variance of χ^2 distribution is twice the degrees of freedom.

iii). If χ^2 is a chi-square variate with ν degrees of freedom, then $\frac{\chi^2}{2}$ is a gamma variate with parameter $\frac{\nu}{2}$

iv). Standard χ^2 variate tends to standard normal variate as $n \rightarrow \infty$.

Applications:

i). To test if the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$.

ii). To test the goodness of fit.

iii). To test the independence of attributes.

iv). To test the homogeneity of independence estimates of the population variance.

---

5. The table below gives the no of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days:	Mon	Tues	Wed	Thurs	Fri	Sat
No of accidents:	14	18	12	11	15	14

Soln.

Given Total No. of Students = 84

No. of days = 6

$\therefore$ Expected frequency $E_i = \frac{84}{6} = 14$.

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
14	14	0	0
18	14	16	1.14
12	14	4	0.285
11	14	9	0.642
15	14	1	0.071
14	14	0	0

$$\sum \frac{(O_i - E_i)^2}{E_i} = 2.1428$$

Step 1:

H_0: The accidents are uniformly distributed

H_1: The accidents are not uniformly distributed

Step 2:

LOS, at $\alpha = 5\%$

Step 3:

Test Statistic:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$= 2.1428$

Step 4:

Degrees of freedom, $\nu = n-1$

$= 6-1$.

$\nu = 5$

$\therefore \chi^2_\alpha = 11.07$

Step 5:

Conclusion:

Calculated value < Table value

$\therefore$ Ho is accepted.

Hence the accidents are uniformly distribut

2. A die was thrown 498 times. Denoting x to be the number appearing on the top face of it, the observed frequency of x is given below.

x :	1	2	3	4	5	6
f :	69	78	85	82	86	98

what opinion you would form for the accuracy of the die?

Soln.

Given Expected frequency

$$E_i = \frac{\text{Total Frequencies}}{6}$$

$$= \frac{498}{6} = 83$$

x	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	69	83	196	2.3614
2	78	83	25	0.3012
3	85	83	4	0.0481
4	82	83	1	0.0120
5	86	83	9	0.1084
6	98	83	225	2.7108

$$\Sigma \frac{(O_i - E_i)^2}{E_i} = 5.5419$$

Step 1:

H_0: A Die is unbiased

H_1: A Die is not unbiased or biased

Step 2:

LOS, at $\alpha = 5\%$

Step 3:

Test Statistic, $\chi^2 = \Sigma \frac{\Sigma (O_i - E_i)^2}{E_i} = 5.542$

Step 4:

Degrees of freedom, $\nu = n-1$

$= 6-1$

$= 5$

$$\chi^2_\alpha = 11.07$$

Step 5:

Conclusion

Calculated Value < Table Value

$\therefore$ H_0 is accepted

i.e., A die is unbiased.

Chi- Square Test for Independence of Attributes :

$$\chi^2 = \frac{(O_i - E_i)^2}{E_i}$$

where $O_i \rightarrow$ observed frequency
$E_i \rightarrow$ Expected frequency

$$E_i = \frac{(\text{row total})(\text{column total})}{\text{whole total}}$$

Degrees of freedom $\nu = (s-1)(t-1)$

Q. On the basis of information noted below, find out whether the new treatment is comparatively superior to the conventional one.

	Favourable	not favourable	Total
New	60	30	90
Conventional	40	70	110
Total	100	100	200

Soln

To find E_i :

$$\frac{90 \times 100}{200} = 45 \qquad \frac{90 \times 100}{200} = 45$$

$$\frac{110 \times 100}{200} = 55 \qquad \frac{110 \times 100}{200} = 55$$

O_i	E_i	$(O_i - E_i)^2/E_i$
60	45	5
30	45	5
40	55	4.09
70	55	4.09

$$\sum \frac{(O_i - E_i)^2}{E_i} = 18.18$$

Step 1:

H_0: There is no difference b/w new & conventional treatment

H_1: There is difference b/w new & conventional treatment.

Step 2:

LOS, $\alpha = 5\%$.

Step 3:

Test Statistic : $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$= 18.18$

Step 4:

Degrees of freedom: $\nu = (s-1) * (t-1)$

$= (2-1 * 2-1)$

$= 1$

$\therefore \chi^2_\alpha = 3.841$

Step 5:

Conclusion :

$|\chi^2| = 18.18 > 3.841 = |\chi^2_\alpha|$

$\therefore H_0$ is rejected at 5% LOS.

i.e., There is difference b/w new & conventional treatment.

2]. Two researchers A & B adopted different techniques while rating the students level. Can you say that the techniques adopted by them are significant?

Researchers	Below Avg.	Avg.	Above Avg	Genius	To
A	40	33	25	2	10
B	86	60	44	10	20
Total	126	93	69	12	30

Soln.

To find E_i :

$\frac{100\times126}{300}=42$ $\frac{100\times93}{300}=31$ $\frac{100\times69}{300}=23$ $\frac{100\times12}{300}=4$

$\frac{200\times126}{300}=84$ $\frac{200\times93}{300}=62$ $\frac{200\times69}{300}=46$ $\frac{200\times12}{300}=8$

O_i	E_i	(O_i-E_i)	$\frac{(O_i-E_i)^2}{E_i}$
40	42	-2	0.0952
33	31	2	0.129
25	23	2	0.173
2	4	-2	1
86	84	2	0.047
60	62	-2	0.064
44	46	-2	0.086
10	8	2	0.5

$$\sum\frac{(O_i-E_i)^2}{E_i} = 2.097$$

Step 1:

H_0: There is no difference b/w the two researchers

H_1: There is difference b/w the two researchers

Step 2:

LOS, $\alpha = 5\%$

Step 3:

Test Statistics: $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$= 2.097$

Step 4:

Degrees of freedom: $\nu = (4-1) * (2-1)$

$= 3 * 1$

$= 3$

$\therefore \chi^2_\alpha = \cancel{2}7.115$

Step 5:

Conclusion:

$\chi^2 = 2.097 < 7.115 = \chi^2_\alpha$

$\therefore$ Ho is accepted

$\therefore$ There is no difference b/w the two researchers.