Maximum Strain Energy Theory:

1) Determine the Diameter of a bolt which is Subjected to an axial pull of 9 kN together with a transverse shear force of 4.5 kN. Elastic Limit in tension is 225 N/mm². Factor of Safety = 3 & Poisson's ratio = 0.3 According to Maximum strain Energy theory.

$P = 9\ kN = 9000\ N$

$F = 4500\ N$

$\sigma_{et} = 225\ N/mm^2$

$F.O.S = 3, \quad \mu = 0.3$

Permissible Simple stress in tension, $\sigma_t = \dfrac{\sigma_{et}}{\text{Safety Factor}}$

$= \dfrac{225}{3} = 75\ N/mm^2$

Other Values, $\sigma = \dfrac{11459}{d^2}\ N/mm^2$

$\tau = \dfrac{5729.5}{d^2}\ N/mm^2$, $d \rightarrow$ Dia of Bolt

The Max & Min principal stresses are

$$\sigma_1 \ \& \ \sigma_2 = \frac{1}{2} \times \sigma \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{11459}{2d^2} \pm \frac{8103}{d^2} \qquad \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{8103}{d^2}$$

$$= \frac{5729.5}{d^2} \pm \frac{8103}{d^2}$$

$$\sigma_1 = \frac{5729.5}{d^2} + \frac{8103}{d^2} = \frac{13882.5}{d^2}\ N/mm^2$$

$$\sigma_2 = \frac{5729.5}{d^2} - \frac{8103}{d^2} = -\frac{2373.5}{d^2} \ N/mm^2$$

Hence, the principal stresses at the point are σ_1, σ_2 & 0.

(or)

$$\frac{13832.5}{d^2}, \ \frac{-2373.5}{d^2}, \ 0$$

According to Max. strain Energy theory.

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \times \sigma_2) = \sigma_t$$

$$\left[\frac{13832.5}{d^2}\right]^2 + \left[\frac{-2373.5}{d^2}\right]^2 - 2\times 0.3\left[\frac{13832.5}{d^2} \times \frac{-2373.5}{d^2}\right] = 75^2$$

$$\frac{21667.25\times 10^4}{d^4} = 5625$$

$$d = \left(\frac{21667.25\times 10^4}{5625}\right)^{1/4}$$

$$= 10\times(3.852)^{1/4}$$

$$= 10\times 1.401 = 14.01 \ mm.$$

Maximum shear strain Energy Theory:

→ Mises - Henky theory.

→ Energy of Distortion theory.

The Failure of the Material occurs when the total shear strain Energy per Unit Volume in the stressed Material reaches a Value equal to shear strain Energy

Per Unit Volume at the elastic Limit in the simple tension.

The total shear strain Energy per Unit Volume due to principal stresses are given by

$$= \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

$\frac{\text{Shear Strain Energy}}{\text{Volume}}$ at elastic Limit in Simple tension will be $= \frac{1}{12C}\left[2\times \sigma_{et}^2\right]$

$$\sigma_t = \frac{\sigma_{et}}{\text{Safety factor}}.$$

For Design purpose, the Eqn is

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2\times(\sigma_t)^2$$

For Two Dimensional stress System $\sigma_3 = 0$

$$\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 = \sigma_t^2.$$

1) The principal stresses at a point in an elastic Material are 200 N/mm² (tensile), 100 N/mm² (tensile) & 50 N/mm² (Compressive). If the stress at the elastic tension is 20 N/mm². Determine whether the failure will occur or not according to Max. Shear strain Energy.

$\sigma_1 = 200\ N/mm^2$

$\sigma_2 = 100\ N/mm^2$.

$\sigma_3 = 50\ N/mm^2$ (Comp) $= -50\ N/mm^2$

Elastic Limit in Simple tension $\sigma_{et} = 200\ N/mm^2$

$\mu = 0.3$.

The total shear strain energy per Unit Volume due to Principal stresses σ_1, σ_2 & σ_3 in the stressed material

$$= \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

$$= \frac{1}{12C}\left[(200-100)^2+\left[100-(-50)\right]^2+\left[-50-200\right]^2\right]$$

$$= \frac{1}{12C} \times 95000 \rightarrow (1)$$

The shear strain energy per Unit Volume at elastic Limit in the Simple tension (here principal stresses are σ_{et}, 0, 0).

$$= \frac{1}{12C}\left[(\sigma_{et}-0)^2+(0-0)^2+(0-\sigma_{et})^2\right]$$

$$= \frac{1}{12C} \times 2 \times (\sigma_{et})^2$$

$$= \frac{1}{12C} \times 2 \times (200)^2 = \frac{1}{12C} \times 80000 \rightarrow (2)$$

Now, applying theory of Max. shear strain Energy.

From (1) & (2)

$$\frac{95000}{12C} > \frac{80000}{12C}$$, Hence, Failure will occur.

2) According to theory of Max. shear strain Energy Determine the diameter of the bolt which is subjected to an axial pull of 9 kN together with transverse shear force of 4.5 kN. Elastic Limit in tension is 225 N/mm^2. F.O.S = 3 & Poisson's Ratio = 0.3

Axial pull $P = 9\ kN = 9000\ N$

Shear Force, $F = 4.5\ kN = 4500\ N$

$\sigma_{et} = 225\ N/mm^2$.

F.O.S = 3.

$$\sigma_t = \frac{\sigma_{et}}{3} = \frac{225}{3} = 75\ N/mm^2.$$

$\mu = 0.3$.

$d \rightarrow$ dia of Bolt in mm.

$\sigma = \frac{P}{A}$ (Tensile stress due to axial pull).

$$= \frac{P}{\pi/4 \times d^2} = \frac{9000 \times 4}{\pi d^2}\ N/mm^2$$

$$= \frac{11459}{d^2}\ N/mm^2.$$

$$\tau = \frac{5729.5}{d^2}\ N/mm^2.$$

Max & Min principal stresses,

$$\sigma_1 = \frac{13832.5}{d^2}\ N/mm^2 \ \& \ \sigma_2 = -\frac{2373.5}{d^2}\ N/mm^2.$$

Third principal stress $\sigma_3 = 0$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2 \times (\sigma_t)^2$$

$$\left[\frac{13832.5}{d^2}-\left(\frac{-2373.5}{d^2}\right)\right]^2+\left[\frac{-2373.5}{d^2}-0\right]^2+$$

$$\left[0-\frac{13832}{d^2}\right]^2=2\times75^2$$

$$\frac{45960.35\times10^4}{d^4}=11250$$

$$d^4=\frac{45960.35\times10^4}{11250}$$

$$d=(4.08536)^{1/4}\times10$$

$$\boxed{d=14.217\text{ mm}}$$