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DEPARTMENT OF AEROSPACEENGINEERING

19ASE306 – THEORY OF VIBRATIONS AND AEROELASTICITY

III YEAR VI SEM

UNIT IV – APPROXIMATE METHODS

TOPIC – Matrix Iteration Method

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Subspace Iteration Method

- Most powerful method for obtaining first few Eigen values/Eigen vectors
- Minimum storage is necessary as the subroutine can be implemented as out-of core solver
- Basic Steps
  – Establish p starting vectors, where p is the number of Eigen values/vectors required P<<n
  – Use simultaneous inverse iteration on ‘p’ vectors and Ritz analysis to extract best Eigen values/vectors
  – After iteration converges, use STRUM sequence check to verify on missing Eigen values

$$[k - \mu m] = [L][D][L]^T$$

- Method is called “Subspace” iteration because it is equivalent to iterating on whole of ‘p’ dimension (rather that n) and not as simultaneous iteration of “p’ individual vectors

- Starting vectors

- Strum sequence property

For better convergence of initial lower eigen values ,it is better if subspace is increased to $q > p$ such that,

$$q = \min(2p, p+8)$$

Smallest eigen value is best approximated than largest value in subspace q.

Starting Vectors

(1) When some masses are zero, for non zero d.o.f have one as vector entry.

$$m = \begin{bmatrix} 0 & & & \\ & 2 & & \\ & & 0 & \\ & & & 1 \end{bmatrix}, \quad \{X\} = \begin{Bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{Bmatrix}$$

(2) Take k_{ii}/m_{ii} ratio .The element that has minimum value will have 1 and rest zero in the starting vector.

$$Diagonal\ [k] = \begin{bmatrix} 3 & & & \\ & 2 & & \\ & & 4 & \\ & & & 8 \end{bmatrix}, \quad [m] = \begin{bmatrix} 2 & & & \\ & 0 & & \\ & & 4 & \\ & & & 1 \end{bmatrix}$$

$$k_{i\&}/m_{i\&} = 3/2, \infty, 1, 8$$

$$\{X\} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

- Starting vectors can be generated by Lanczos algorithm- converges fast.
- In dynamic optimisation , where structure is modified previous vectors could be good starting values.

Eigen value problem

$$[k][\phi] = [\Omega][m][\phi] \qquad (1)$$

$$[k]_{n \times p},\ [\phi]_{n \times p}$$

$$[\phi]^T[k][\phi] = [\Omega]_{p \times p} \qquad (2)$$

$$[\phi]^T[m][\phi] = [I] \qquad (3)$$

Eqn. 2 are not true. Eigen values unless P = n

If [ϕ] satisfies (2) and (3),they cannot be said that they are true Eigen vectors. If [ϕ] satisfies (1),then they are true Eigen vectors.

Since we have reduced the space from n to p. It is only necessary that subspace of ‘P’ as a whole converge and not individual vectors.

Algorithm:

Pick starting vector X_R of size n x p

For k=1,2,…..

$$[k][\bar{X}_{k+1}] = [m]\{\bar{X}_k\}$$ static

$$[k]_{k+1} = \{\bar{X}\}^T_{k+1}[k]\{\bar{X}_{k+1}\}$$ p x p

Smaller eigen value problem, Jacobi

$$[m]_{k+1} = \{\bar{X}\}^T_{k+1}[m]\{\bar{X}_{k+1}\}$$ p x p

$$[k]_{k+1}\{Q\}_{k+1} = [m]_{k+1}\{Q_{k+1}\}[\Lambda]_{k+1}$$

$$[X]_{k+1} = \{\bar{X}\}_{k+1}[Q]_{k+1}$$

$\Lambda_{k+1} \rightarrow \Lambda$ $\{X\}_{k+1} \rightarrow \phi$ $k \rightarrow \infty$

QUESTIONS RELATED ABOVE SLIDES

Factorization	$[k]=[L][D][L]^T$	(1/2)nm^2 + (3/2)nm
Subspace Iteration	$[k][\bar{X}]_{k+1}=[Y]_k$	nq(2m+1)
	$[k]_{k+1}=[\bar{X}]^T_{k+1}[Y_k]$	(nq/2)(q+1)
	$[M_{k+1}]=[\bar{X}_{k+1}]^T[\bar{Y}_{k+1}]$	(nq/2)(q+1)
	$[k]_{k+1}[Q]_{k+1}=[M]_{k+1}[Q]_{k+1}[\quad]_{k+1}$	
	$[Y]_{k+1}=[\bar{Y}]_{k+1}[Q]_{k+1}$	nq^2
Sturm sequence check	$[\bar{k}]=[k]-\mu[M]$	n(m+1)
	$[\bar{k}]=[L][D][L]^T$	(1/2)nm^2 + (3/2)nm
	$\dfrac{\left\| [k][\phi]_i^{k+1}-\lambda_i^{k+1}[M][\phi_i]^{k+1} \right\|}{\left\| [k][\phi_i]^{k+1} \right\|}$	4nm + 5n

Total for p lowest vector.

@ 10 iteration with $nm^2 + nm(4+4p)+5np$

q = min(2p , p+8) is $20np(2m+q+3/2)$

↓

This factor increases as that iteration increases.

N = 70000,b = 1000, p = 100, q = 108   Time = 17 hours

Aim: Generate (neq x m) modal matrix (Ritz vector).

- Find λ_k and $\{ u \}_k$ for the k^{th} component

Let $[\phi]_k$ = substructure Modal matrix

which is nk x nϕ , nk = # of interior d.o.f

nϕ = # of normal modes take determined for that structure

Assuming 'I' structure,

(2)

$$[R] = \begin{bmatrix} [\phi]_1 & [0] & [0] & . & . & . \\ [0] & [I]_{1,2} & [0] & . & . & . \\ [\phi]_2 & [0] & [0] & . & . & . \\ [0] & [0] & [I]_{2,3} & . & . & . \\ . & . & . & . & . & . \\ [\phi]_l & [0] & [0] & . & . & . \end{bmatrix}$$

Neq x m

$[I]_{k,k+1}$ - with # of rows = # of attachment d.o.f. between k and k+1

= # of columns

Ritz analysis:

Determine $[K_r] = [R]^T [k] [R]$

$[M_r] = [R]^T [M] [R]$

$[k_r] \{X\} = [M]_r + [X] [\quad]$ - Reduced Eigen value problem

Eigen vector Matrix, $[\phi] = [R] [X]$

Example

Use the subspace Iteration to calculate the eigen pairs (λ_1,ϕ_1) and (λ_2,ϕ_2) of the problem $K\phi = \lambda M \phi$,where

$$K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}; \quad M = \begin{bmatrix} 0 & & & \\ & 2 & & \\ & & 0 & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \bar{X}_2 = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{X}_2 = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 4 & 3 \\ 4 & 4 \end{bmatrix} \quad and$$

$$K_2 = 4\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}; \quad M_2 = 8\begin{bmatrix} 6 & 4 \\ 4 & 3 \end{bmatrix}$$

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THANK YOU