SNS COLLEGE OF TECHNOLOGY

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DEPARTMENT OF AEROSPACEENGINEERING

19ASE306 – THEORY OF VIBRATIONS AND AEROELASTICITY

III YEAR VI SEM

UNIT IV – APPROXIMATE METHODS

TOPIC – Dunkerley’s Method

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Dunkerley's Method

$$\{x\} = [d]\{F\}$$

Equation of motion : $[d][m]\{\ddot{x}\} + \{x\} = \{0\}$

Let [m] be the diagonal matrix, $[d][m]\{\ddot{x}\} + \{x\} = \{0\}$

$$\left[[d][m] - \frac{1}{p^2}[I]\right]\{x\} = \{0\}$$

set

Det
$$\begin{vmatrix} d_{11}m_1 - \frac{1}{p^2} & d_{12}m_2 & . & . & d_{1n}m_n \\ d_{21}m_1 & d_{22}m_2 - \frac{1}{p^2} & . & . & d_{2n}m_n \\ . & . & . & . & . \\ . & . & . & . & . \\ d_{n1} & d_{n2}m_2 & . & . & d_{nn}m_n - \frac{1}{p^2} \end{vmatrix} = 0$$

Given a n^{th} order polynomial equation, $(1/p^2)$

$$\left(\frac{1}{p^2}\right)^n + (d_{11}m_1 + d_{22}m_2 + \ldots + d_{nn}m_n)\left(\frac{1}{p^2}\right)^{n-1} + \ldots = 0$$

Sum of the roots of characteristic equation,

$$\left[\frac{1}{p_1^2} + \frac{1}{p_2^2} + \ldots \frac{1}{p_n^2}\right] = (d_{11}m_1 + d_{22}m_2 + \ldots + d_{nn}m_n)$$

$$let \qquad p_{ii} = \sqrt{\frac{k_{ii}}{m_i}} \qquad \text{where} \quad k_{ii} = \frac{1}{d_{ii}}$$

d_{ii} is the flexibility coefficient equal to deflection at i resulting from a unit load of i, its reciprocal must be the stiffness coefficient k_{ii}, equal to the force per unit deflection at i.

The estimate to the fundamental frequency is made by recognizing p_2 ,p_3 etc are natural frequencies of higher modes and larger than p_1.

By neglecting these terms($1/p_2^2$, …$1/p_n^2$) ,$1/p_1^2$ is larger than its true value and there fore p_1 is smaller than the exact value of the fundamental frequency

$$\frac{1}{p_1^2} \leq \frac{1}{p_{11}^2} + \frac{1}{p_{22}^2} + \ldots\ldots + \frac{1}{p_{nn}^2}$$

$$\frac{1}{p_1^2} \; ; \; \frac{1}{p_{11}^2} + \frac{1}{p_{22}^2} + \ldots\ldots + \frac{1}{p_{nn}^2}$$

Dunkerley's Approximation

It provides a lower bound estimate for the fundamental frequency.

Let p = natural frequency of system

$p_A, p_B, p_C, \ldots\ldots.. p_N$ = exact frequencies of component systems

Then

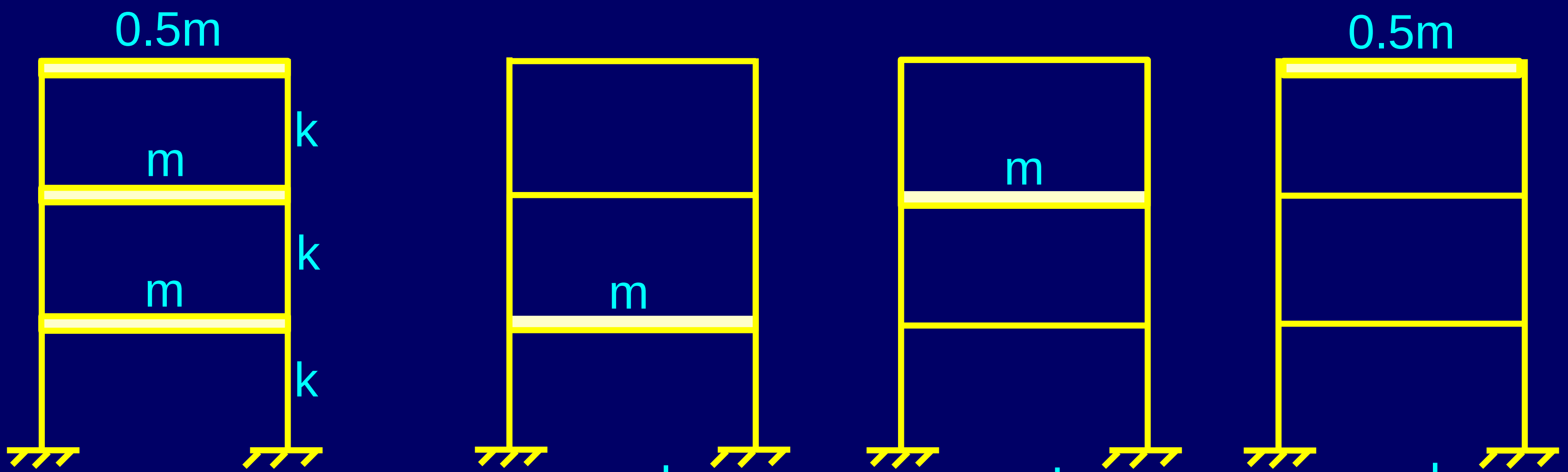
$$\frac{1}{p^2} ; \frac{1}{p_A^2} + \frac{1}{p_B^2} + \frac{1}{p_C^2} + \ldots\ldots\ldots + \frac{1}{p_N^2}$$

or

$$T^2 ; T_A^2 + T_B^2 + T_C^2 + \ldots\ldots\ldots\ldots\ldots + T_N^2$$

The frequency so determined can be shown to be lower than the exact.

Example # 1



$$p_A^2 = \frac{k}{m} \qquad p_B^2 = \frac{k}{2m} \qquad p_C^2 = \frac{k}{3m/2}$$

$$\frac{1}{p^2} = \frac{m}{k}[1+2+1.5] = 4.5\frac{m}{k}$$

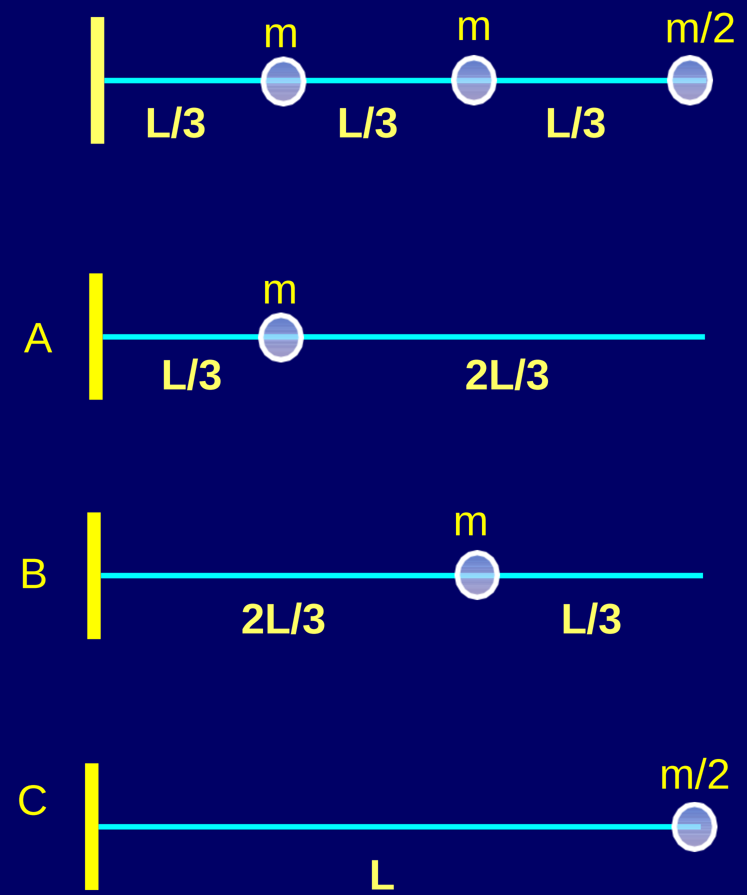
$$p^2 \approx \frac{k}{4.5m} = 0.2222\frac{k}{m}$$

If natural modes of component systems A, B, C are close of each other, then the value of p determined by this procedure can be shown to be close to the exact.

Example # 2

Consider the cantilever beam shown for which the component systems A,B,C are indicated .

Since the natural modes of the system are in closer agreement in this case than for the system of the shear beam type considered in the previous example, the natural frequency computed by Dunkereley's method can be expected to be closer to the exact value than with case before.



$$p_A^2 = 3\frac{EI}{(L/3)^3\, m} = 81\frac{EI}{mL^3}$$

$$p_B^2 = 3\frac{EI}{(2L/3)^3\, m} = \frac{81}{8}\frac{EI}{mL^3}$$

$$p_C^2 = 3\frac{EI}{L^3\,(m/2)} = 6\frac{EI}{mL^3}$$

$$\frac{1}{p^2} \approx \frac{1}{p_A^2} + \frac{1}{p_B^2} + \frac{1}{p_C^2} = \left(\frac{1}{81} + \frac{8}{81} + \frac{1}{6}\right)\frac{mL^3}{EI}$$

$$\frac{1}{p^2} \approx \left(\frac{1}{9} + \frac{1}{6}\right)\frac{mL^3}{EI} = \frac{15}{54}\frac{mL^3}{EI}$$

$$p^2 \approx 3.6\frac{EI}{mL^3} \quad (Low\ bound)$$

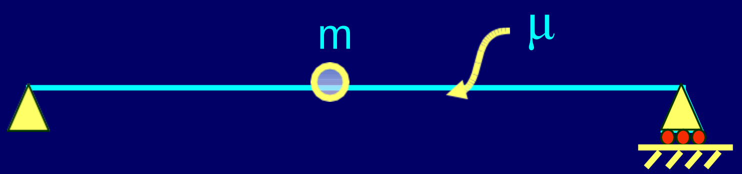
$$(p_{exact}^2 = 3.7312\frac{EI}{mL^3})$$

As expected the agreement is excellent in this case.

Example # 3

Upper bound: Determined by Rayleigh’s method with $y(x) = y_0 \sin(\pi x/L)$ is,

$$p^2 \approx \frac{\pi^4 (EI/L^3)}{\mu L + 2m}$$



Lower bound: Determined by Dunkerley’s approximation

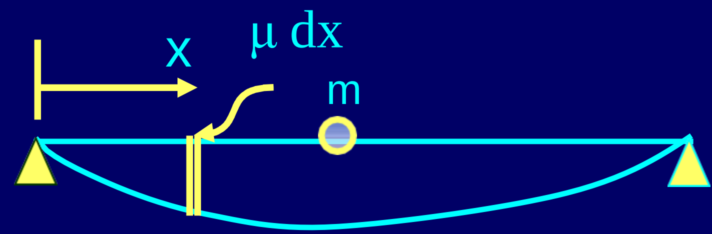
If we consider one mode,

$$p_A^2 = \frac{\pi^4 EI}{\mu L^4}; \qquad p_B^2 = \frac{48EI}{mL^3}$$

$$\frac{1}{p^2} = \frac{\mu L^4}{\pi^4 EI} + \frac{mL^3}{48EI} = \left[\mu L + \frac{\pi^4 m}{48}\right]\frac{L^3}{\pi^4 EI}$$

$$where \qquad p^2 \approx \frac{(EI/L^3)}{\left(\frac{\mu L}{90} + \frac{\pi^4 m}{48}\right)}$$

$$For \quad m = \mu L, we \ find$$



$$\int_0^l \frac{\mu x^2 (l-x)^2 dx}{3lEI} + \frac{ml^3}{48EI} = \frac{\mu}{3lEI}\left[l^2 l^3 - 2l l^4 + \frac{l^5}{5}\right] + \frac{ml^3}{48EI}$$

$$= \frac{\mu l^4}{3EI}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] + \frac{ml^3}{48EI} = \frac{\mu l^4}{90EI} + \frac{ml^3}{48EI}$$

$$\frac{\pi^4}{3.117}\frac{EI}{\mu L^4} \le p_1^2 \le \frac{\pi^4}{3}\frac{EI}{\mu L^4}$$

Consider all modes,

$$p_1^2 = \frac{\mu l^4}{\pi^4 EI}\left[\sum_{n=1}^{\infty}\frac{1}{n^4}\right] + \frac{ml^3}{48EI}$$

$$p_1^2 = \frac{\mu l^4}{\pi^4 EI}\left[\frac{\pi^4}{90}\right] + \frac{ml^3}{48EI}$$

Limitation of procedures:

- One cannot improve the accuracy of the solution (depends on the deflected shape of structure) in a systematic manner.
- Extension of procedure : Rayleigh - Ritz

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THANK YOU