SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF AEROSPACEENGINEERING

19ASE306 – THEORY OF VIBRATIONS AND AEROELASTICITY

III YEAR VI SEM

UNIT IV – APPROXIMATE METHODS

TOPIC – RAYLEIGH'S Method

NAME: Mr.N.Venkatesh., M.Tech
Assistant Professor
Aeronautical Engineering
SNS College of Technology

Rayleigh Method

- This method predicts the fundamental (lowest) natural frequency
- This method based on energy method

$$T = \frac{1}{2} m \dot{x}^2 \longrightarrow T = \frac{1}{2} \{\dot{x}\}^T [M] \{\dot{x}\}$$

$$V = \frac{1}{2} k x^2 \longrightarrow V = \frac{1}{2} \{x\}^T [K] \{x\}$$

Rayleigh Quotient

$$T = \frac{1}{2}\{\dot{x}\}^T [M]\{\dot{x}\} \qquad V = \frac{1}{2}\{x\}^T [K]\{x\}$$

$$\{x\} = \{X\}\sin(\omega t)$$

$$\{\dot{x}\} = -\{X\}\omega\cos(\omega t)$$

$$T_{\max} = \frac{1}{2}\{X\}^T [M]\{X\}\omega^2 \qquad V_{\max} = \frac{1}{2}\{X\}^T [K]\{X\}$$

$$T_{\max} = V_{\max}$$

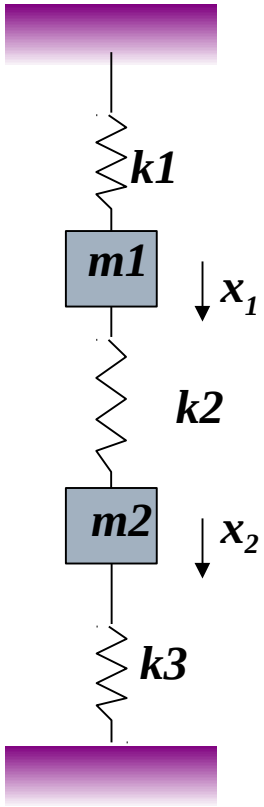
$$\omega^2 = \frac{\{X\}^T [K]\{X\}}{\{X\}^T [M]\{X\}}$$

Rayleigh Method (Calculation procedures)

- Identify [K] and [M]
- Select any trial vector mode {X}
- Predict the fundamental natural frequency based on the Rayleigh Quotient

$$\omega^2 = \frac{\{X\}^T [K] \{X\}}{\{X\}^T [M] \{X\}}$$

Example problem



Predict the fundamental natural frequency using Rayleigh method

k1=10N/m k2=20N/m k3=15N/m

m1 = 1.2 kg m2 = 2.7 kg

$$K := \begin{pmatrix} 30 & -20 \\ -20 & 35 \end{pmatrix} \qquad M := \begin{pmatrix} 1.2 & 0 \\ 0 & 2.7 \end{pmatrix}$$

vector trial $\quad X := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\omega sq := \frac{\left(X^T \cdot K \cdot X\right)}{\left(X^T \cdot M \cdot X\right)}$$

$\omega sq = 6.41$

$\omega n := \sqrt{\omega sq} \qquad \omega n = 2.532$

REFERENCE LINKS

1. Fisher, J.W. (1970) Design of composite beams with formed metal deck. Eng. J. Amer. Inst. Steel Constr., 7, July, 88–96.

2. Wang, Y.C. (2002) Steel and Composite Structures – Analysis and Design for Fire Safety. Spon, London.

3. British Standards Institution BS EN 1994. Design of composite steel and concrete structures. Part 1-1, General rules and rules for buildings. To be published, British Standards Institution, London.

THANK YOU